Equilibrium and Stability of Rectangular Liquid-Filled Vessels

TEXAS A&M ENGINEERING

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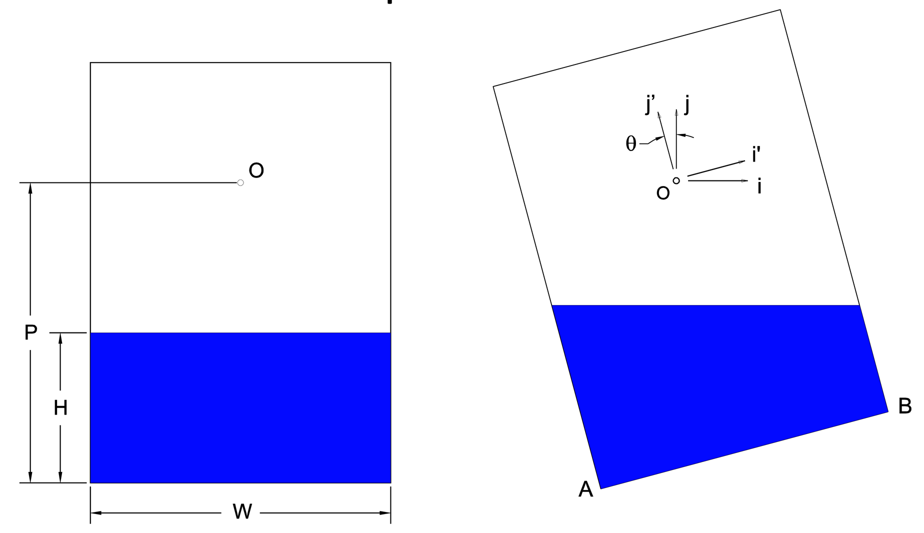
Problem Statement

A rectangular vessel containing liquid can have several rotated equilibrium positions. The location and stability of these equilibria are based on the width of the vessel W, the height of the pivot point P, and the amount of liquid H.

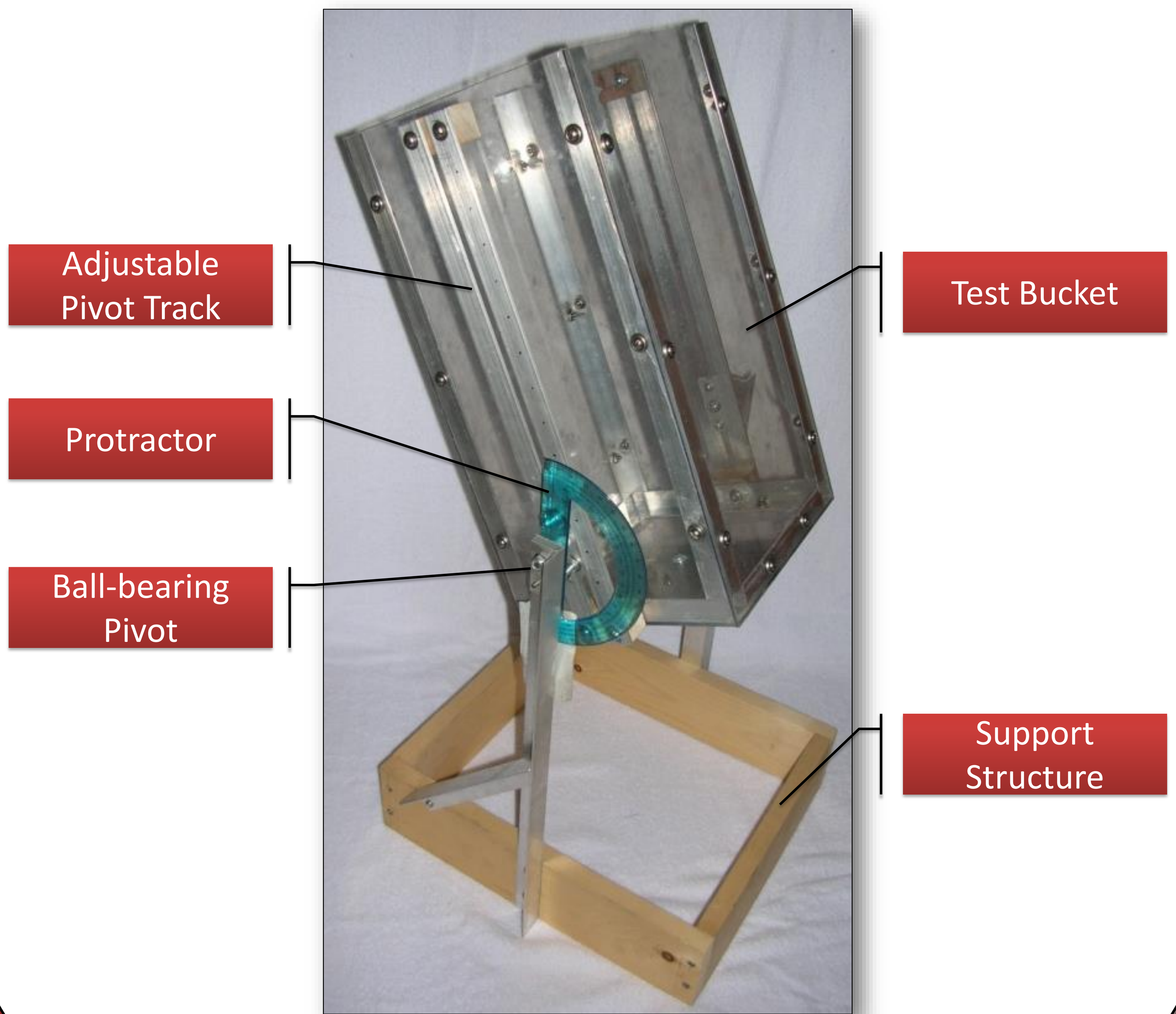


Applications include:
- •Marine structures
- •Tanker ships
- •Aircraft fuel systems
- •Mobile machinery

Ship listing due to shifting liquid payload

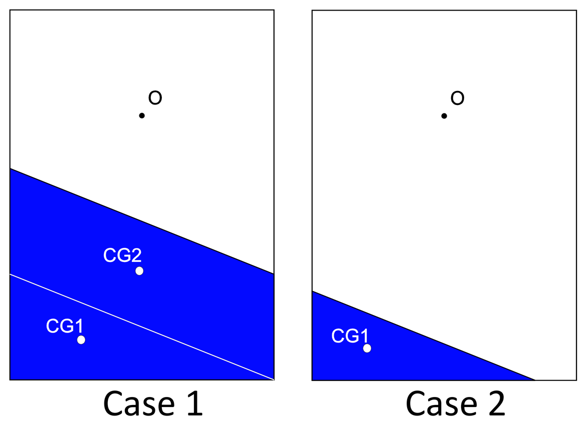
Experiment

A test “bucket” was constructed to validate the calculated results. The data collected supports the predictions.



Solution Method

Define the position of the center of gravity (CG) of the liquid:

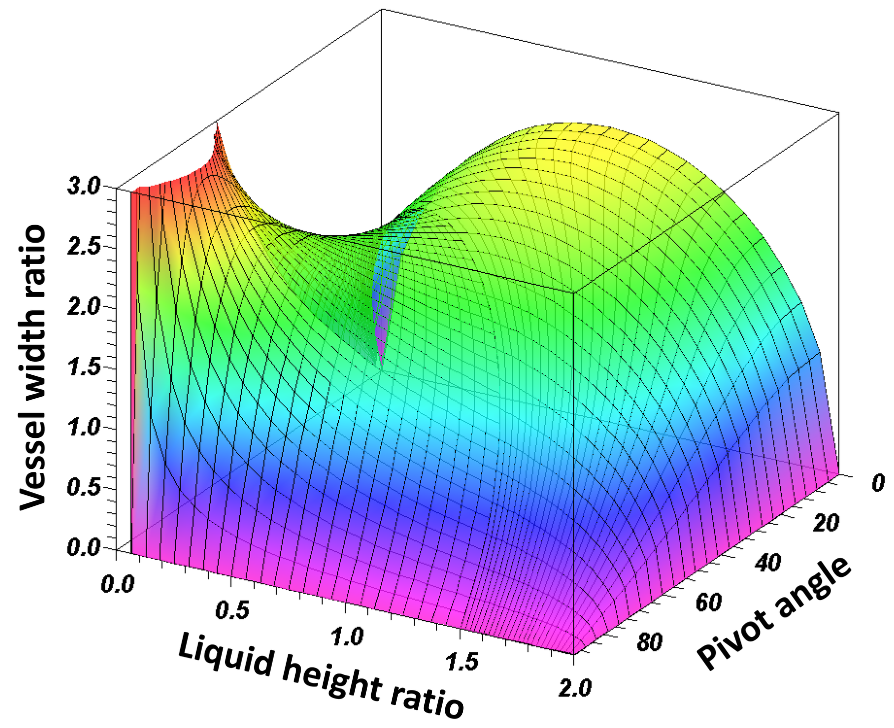


Define non-dimensional gravitational potential of the CG for both cases:

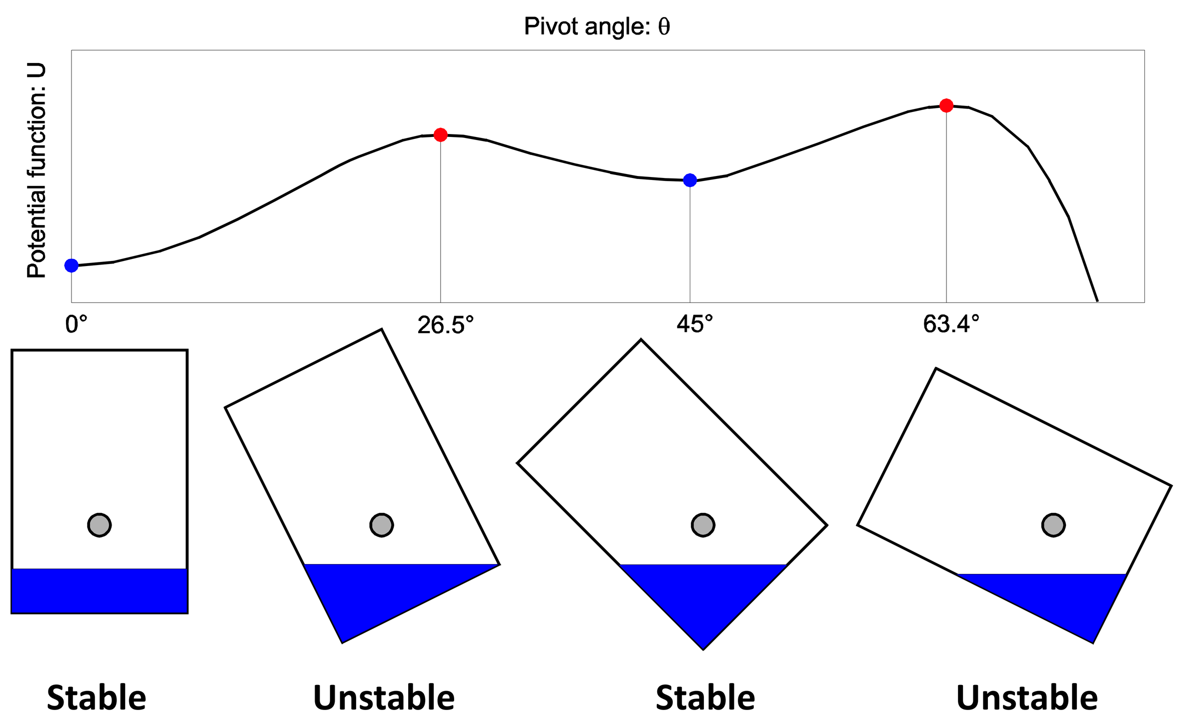
$$U \equiv \vec{r}_{CG} \cdot \vec{j} = \begin{cases} \left(1 - \frac{W^2 \tan^2\theta}{24HP} - \frac{W^2}{12HP} - \frac{H}{2P}\right)\sin\theta & \tan\theta < \frac{2h}{w} \\ \left(\frac{\sqrt{2HW\tan\theta}}{3P} - 1\right)\cos\theta + \left(\frac{\sqrt{2HW}}{3P\sqrt{\tan\theta}} - \frac{W}{2P}\right) & \tan\theta \geq \frac{2h}{w} \end{cases}$$

Derivative zeros correspond to equilibria:

Equilibrium Positions



Second derivative classifies the stability of an equilibrium:

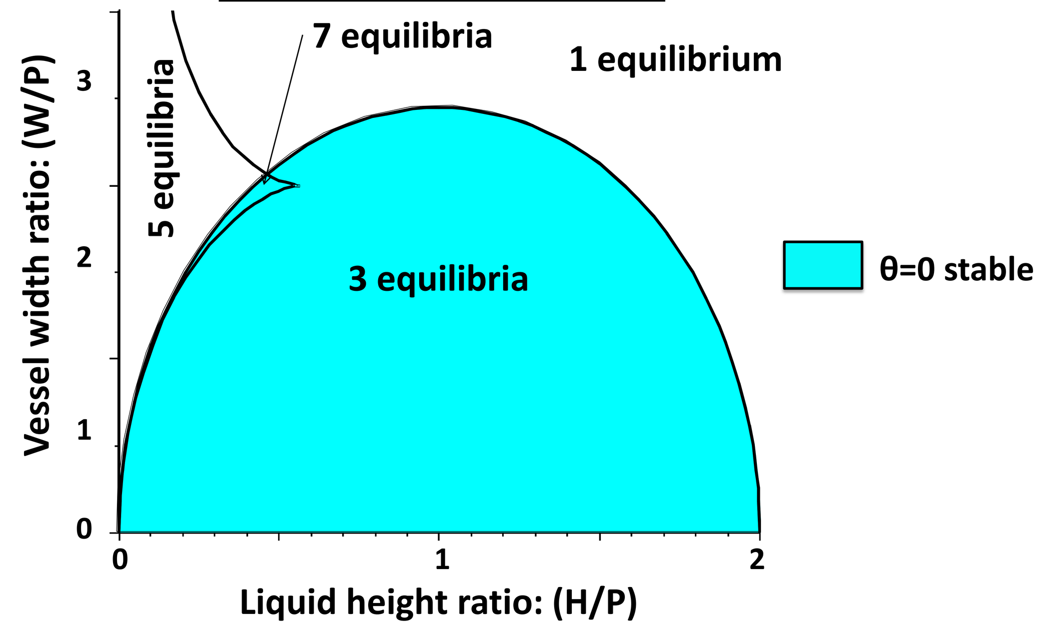


The equations governing Case 1 and Case 2 equilibrium and stability have domains of validity. These restrictions define regions with various numbers of equilibria. Some regions have one equilibrium while other can have up to seven.

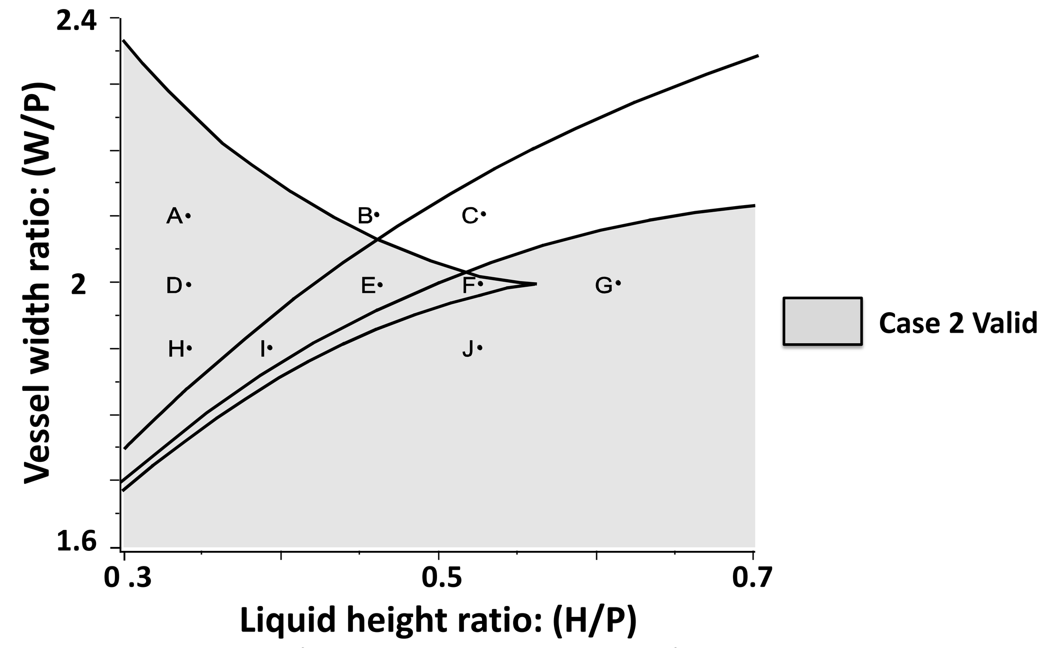
Results

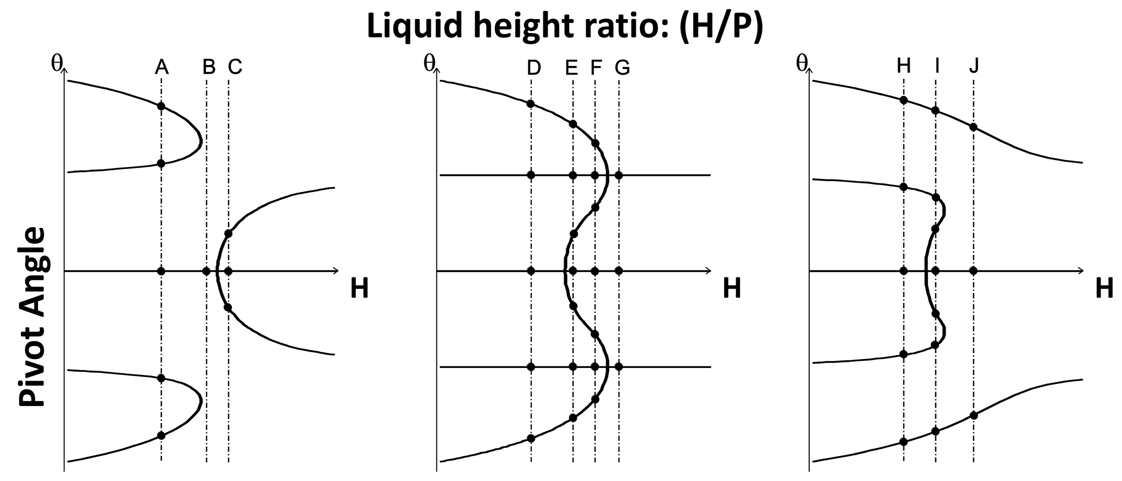
- The vessel can have up to seven equilibrium positions.
- For some amounts of liquid, there are no stable equilibria.
- There is a maximum width of the vessel and maximum height of water for which stable equilibria can exist.

Number of Equilibria



Unfolding of Bifurcation Diagrams





Theoretical and Experimental Bifurcation Diagrams