Mitigating the Effect of Noise in Iterative Projection Phase Retrieval

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Outline

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- Introduction
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  - 2d projection visualization method
- Filtering of Modulus Noise
  - Noise model
  - Observed error behavior
  - Theoretical effects
  - New Fourier Domain Projection
- Resulting Performance
  - 2d projection visualization examples
  - Low SNR example
- Conclusion

Phase Retrieval Problem Statement

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- Objective is to recover the image $g(x, y)$ which has the discrete Fourier transform $G(u, v)$.
- Given the measured Fourier modulus $|G(u, v)|$, estimate the phase of $G(u, v)$ so it can inverted to recover $g(x, y)$.
- At each iteration perform several projections onto constrained domains.

Historically, there is not much attention paid to noise in the measured Fourier modulus noise. Can it be filtered?

R. Trahan and D. Hyland, "Mitigating the effect of noise in the hybrid input-output method of phase retrieval." Applied Optics, Vol. 52, Issue 13, pp. 3031-3037 (2013)

Typical Projection Operators

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- Fourier domain projection preserves phase and constrains modulus to $|F(u,v)|$

$$P_m g(x,y) = \mathrm{FT}^{-1}\left[|F(u,v)|\exp\left(i\varphi(\mathrm{FT}[g(x,y)])\right)\right]$$

- Image domain projection preserves pixel value magnitudes within foreground and zeros others

$$P_s g(x,y) = \begin{cases} |g(x,y)|, & (x,y) \in \gamma(x,y) \\ 0, & \text{otherwise} \end{cases}$$

- Reflections are defined as $R \equiv 2P - I$.

Phase Retrieval Methods

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Algorithm	Iteration
Error Reduction (ER)	$g_{k+1} = P_s P_m g_k$
Solvent Flipping (SF)	$g_{k+1} = R_s P_m g_k$
Hybrid Input-Output (HIO)	$g_{k+1} = \begin{cases} P_m g_k, & (x,y) \in \gamma(x,y) \\ (I - \beta P_m) g_k, & \text{otherwise} \end{cases}$
Difference Map (DM)	$g_{k+1} = \{I + \beta P_s[(1+\gamma_s)P_m - \gamma_s I] - \beta P_m[(1+\gamma_m)P_s - \gamma_m I]\} \mathrm{g}_\mathrm{k}$
Averaged Successive Reflection (ASR)	$g_{k+1} = \frac{1}{2}[R_s R_m + I] g_k$
Hybrid Projection Reflection (HPR)	$g_{k+1} = \frac{1}{2}[R_s(R_m + (\beta - 1)P_m) + I + (1-\beta)P_m] g_k$
Random Averaged Alternating Reflector (RAAR)	$g_{k+1} = \left[\frac{1}{2}\beta(R_s R_m + I) + (1-\beta)P_m\right] g_k$
Levi-Stark method (LS)	$G_{k+1} = (1-\lambda)G_k + FT(\lambda P_s P_m g_k)$

Domain Intersection Analog

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- Each projection imposes a constraint.
  - Image resides in the Support Domain (S)
  - Image Fourier modulus resides in the Fourier modulus domain (M)
- Upon convergence none of the constraints affect the data, i.e. the image is a member of both domains simultaneously.
- Phase retrieval problem has an analog: find the intersection of two non-convex domains.
- The analog can be formulated as a two dimensional problem for easy analysis.

2D Visualization Method

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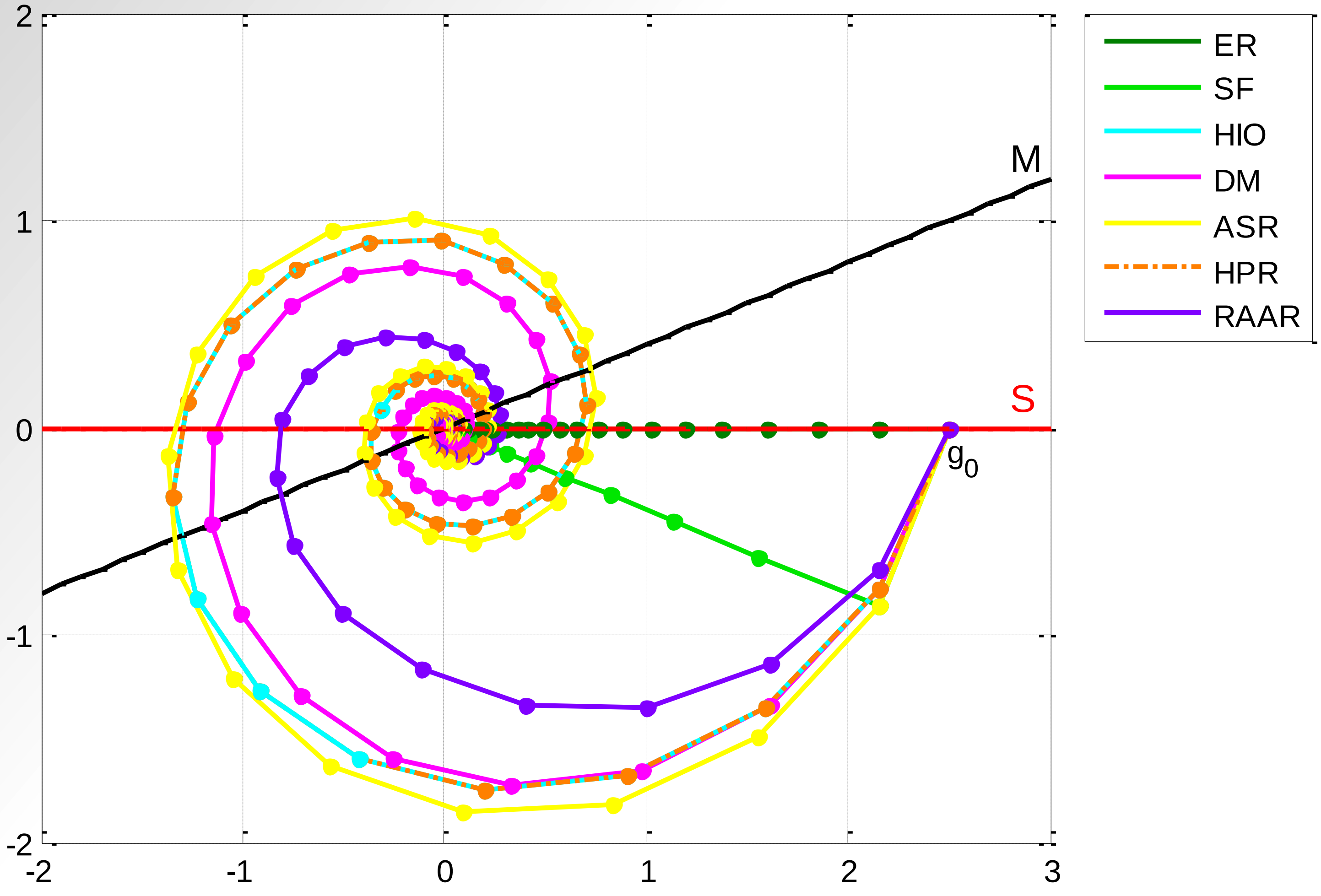

Noise Model

- Noise can be modeled as
$$|F(\pm u, \pm v)| = |F_{True}(u, v)(1 + N[0, \sigma] + N[0, \sigma]i)|$$
- Here the quantity σ is called the Noise-To-Signal Level.
- Model is borrowed from intensity interferometry theory.
- Any other noise model can be considered without changing the rest of this analysis.

Effect of Noise on the HIO

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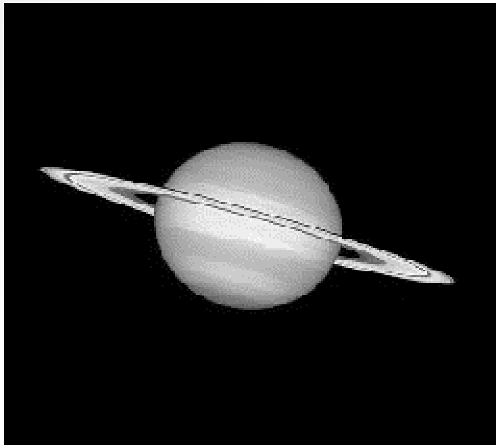
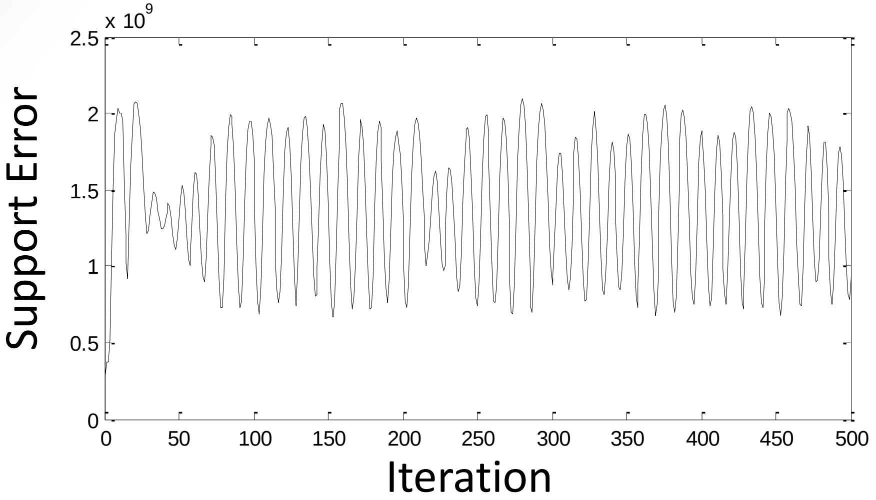
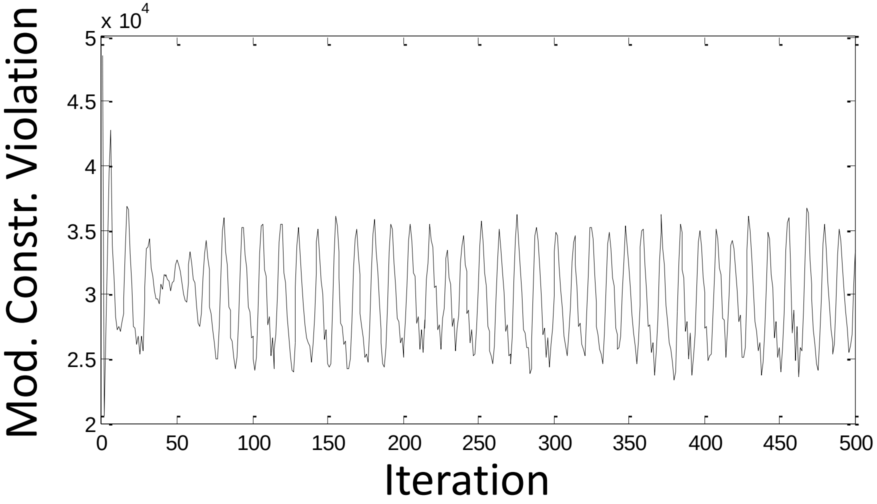
- Oscillation occurs in both image and modulus constraint violations.
- Oscillations tend to be out of phase.
- Alternation in semi-satisfied constraint.







Analysis of the Effect of Noise

- Consider the simple Error-Reduction method, $g_{k+1} = P_s P_m g_k$.
- Let each (u, v) pixel have a variation Δ from its proper value.
- Noisy modulus is
  $\tilde{G}(u, v) = F_{True}(u, v) + \sum_{(a,b)} \Delta(a, b)\delta(u - a, v - b)$
- By linearity of the DFT, gain insight by looking at one noise component, i.e.
  $$G'(u, v) = \Delta\delta(u - a, v - b).$$
- By going through a phase retrieval iteration, the effect of Δ should be eliminated.

Analysis of the Effect of Noise (2)

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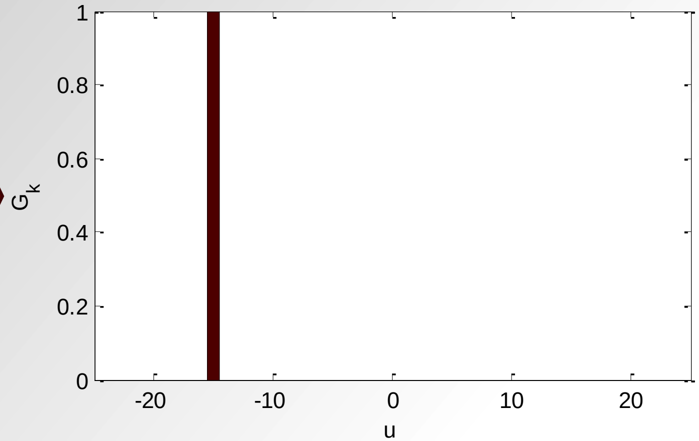
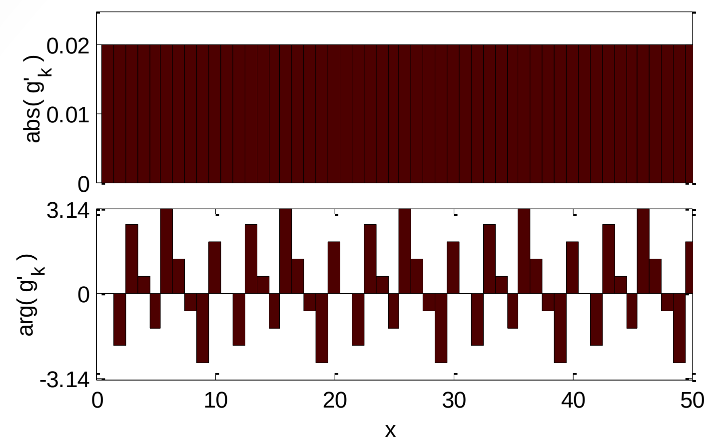
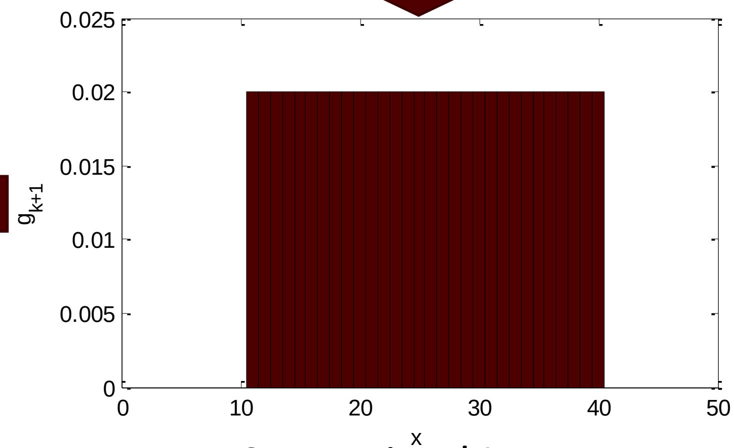
Start



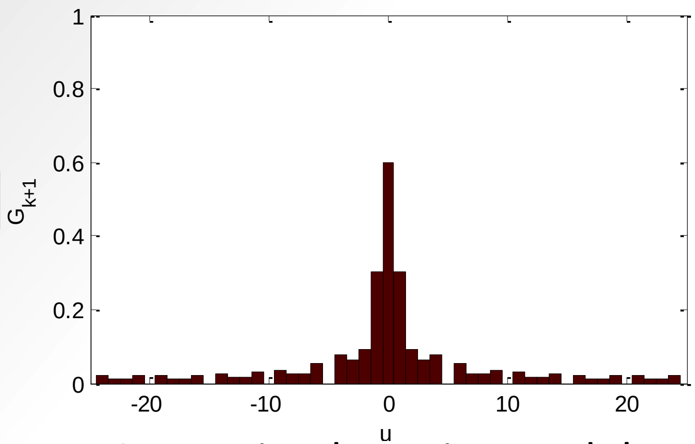
Initial Fourier Modulus



Unconstrained Image



Constrained Image



Constrained Fourier Modulus

Finish

Analysis of the Effect of Noise (3)

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- After one iteration
  - Maximum: $G(0) = \Delta \frac{M-2A}{M}$
  - Frobeniun norm: $\sum_{u=0}^{M-1} |G(u)|^2 = \Delta \frac{M-2A}{M}$
- Filtering is proportional to oversampling.
- This leads to the idea that the modulus should be constrained to a mix of the previous iteration's modulus and the measured modulus.

New Fourier Modulus Projection

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- Modulus should be constrained to a mix of the measured and previous iteration's modulus.
- New projection operator:

$$\tilde{P}_m g(x,y) = \mathrm{FT}^{-1}\left[\hat{F}(u,v)\exp\left(i\varphi(\mathrm{FT}[g(x,y)])\right)\right]$$

$$\hat{F}(u,v) = (1-\lambda)|F(u,v)| + \lambda|\mathrm{FT}[g(x,y)]|$$

- Typical relaxation parameter values:
  - To mitigate constraint oscillation: 0.01
  - To filter noise: 0.9

Intersection / No Relaxation

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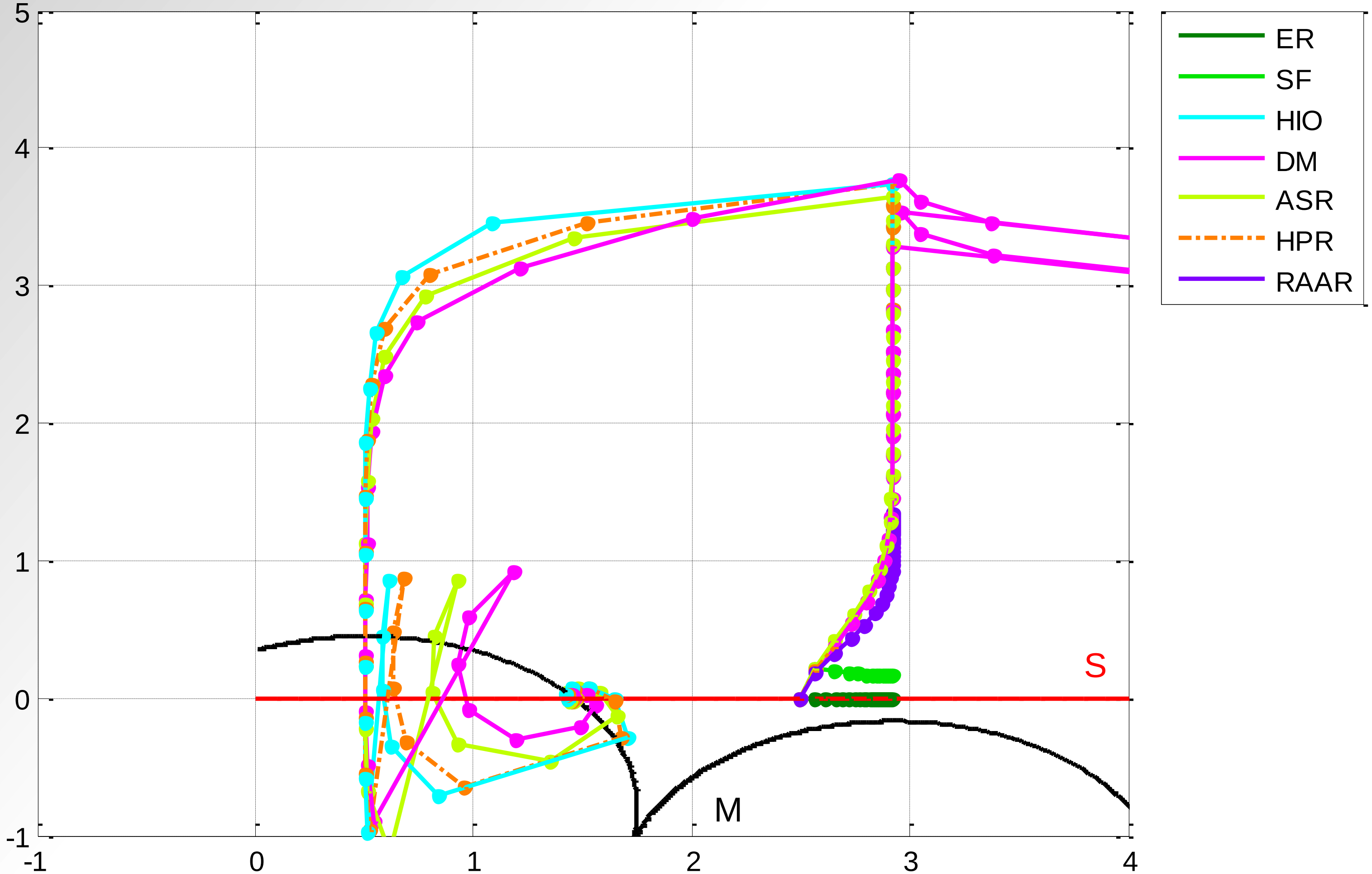

Relaxation after Iteration 30

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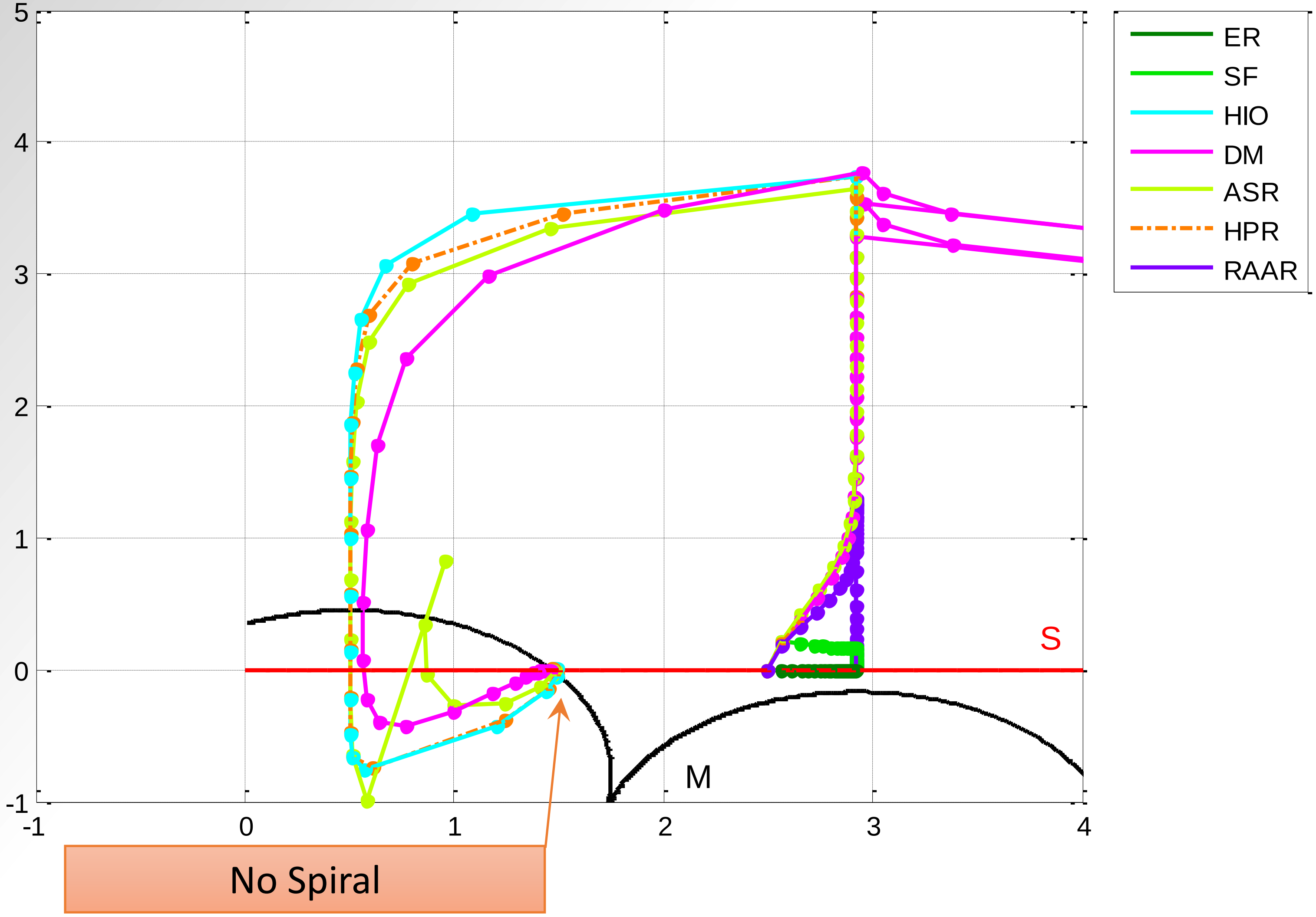

No Intersection / No Relaxation

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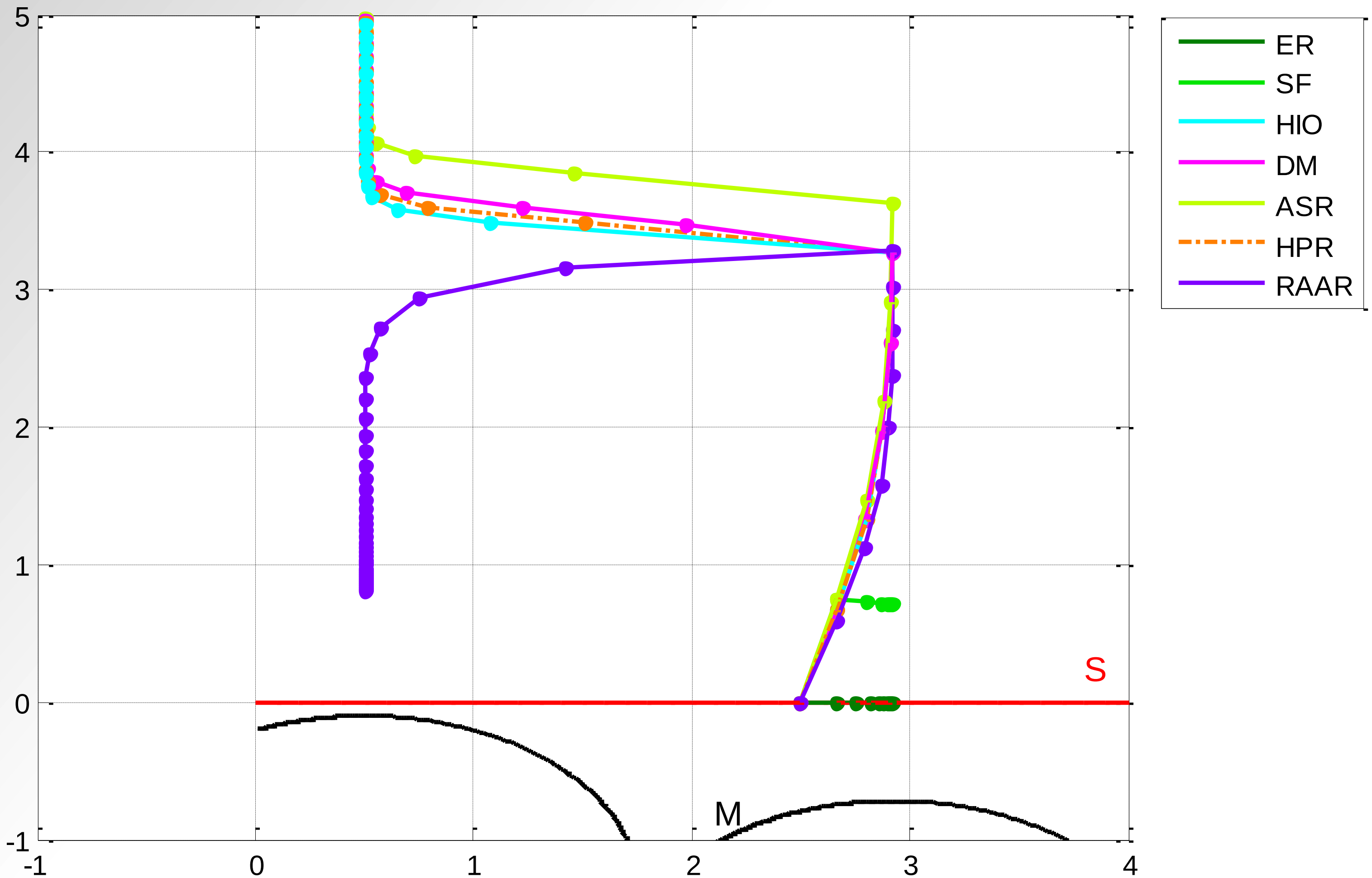

Relaxation after Iteration 10

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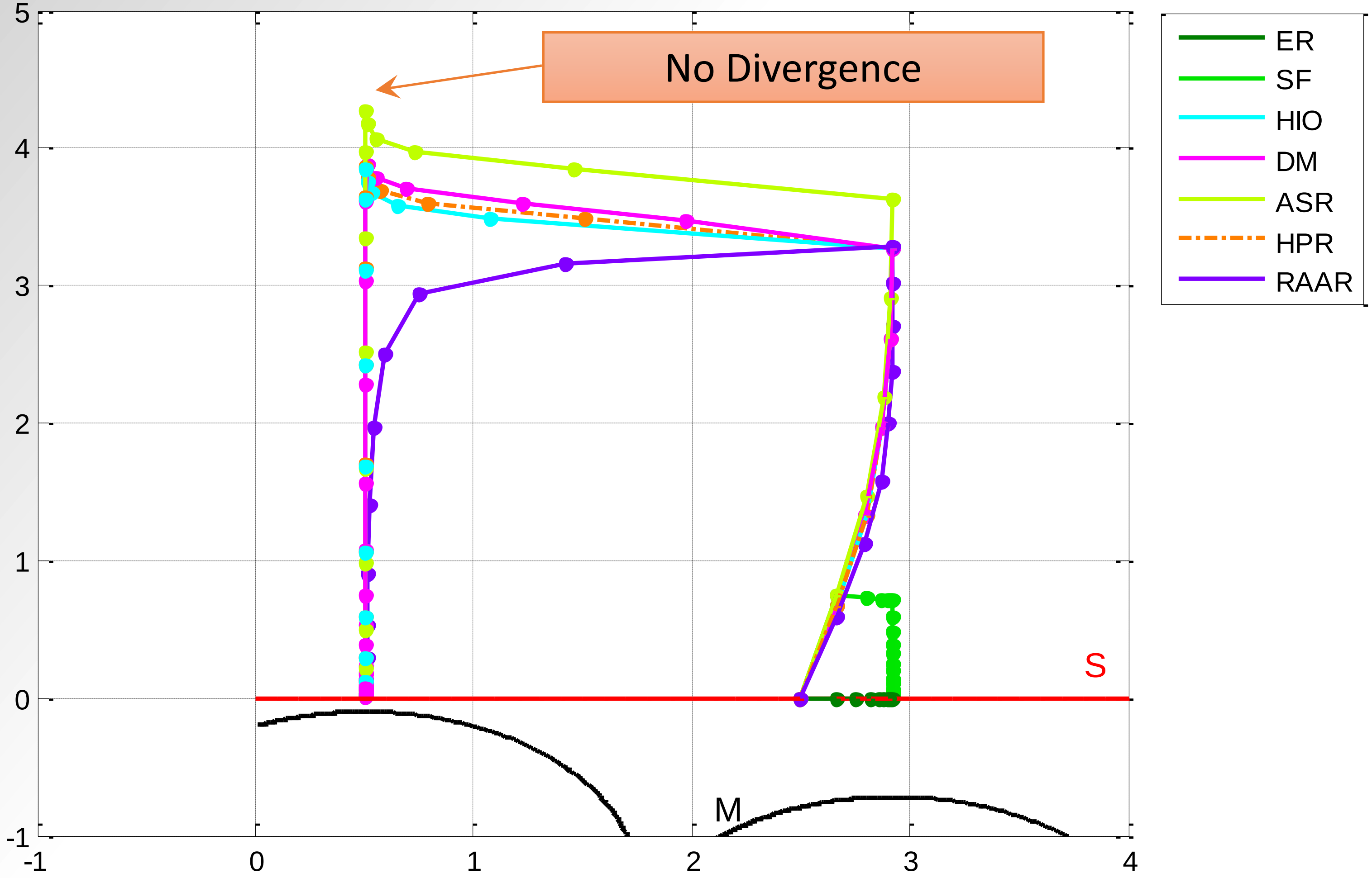

HIO+Relaxation

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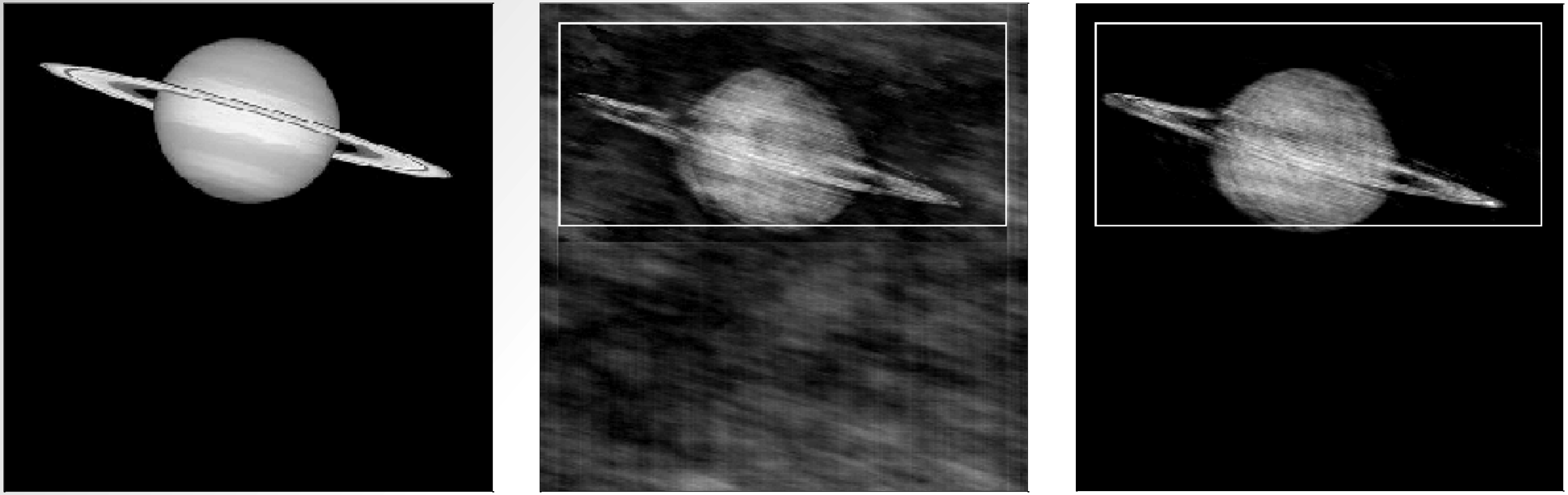

True Image

Iteration 500
No relaxation yet

Iteration 1000
Relaxed Modulus Constraint

HIO+Relaxation Error Metrics

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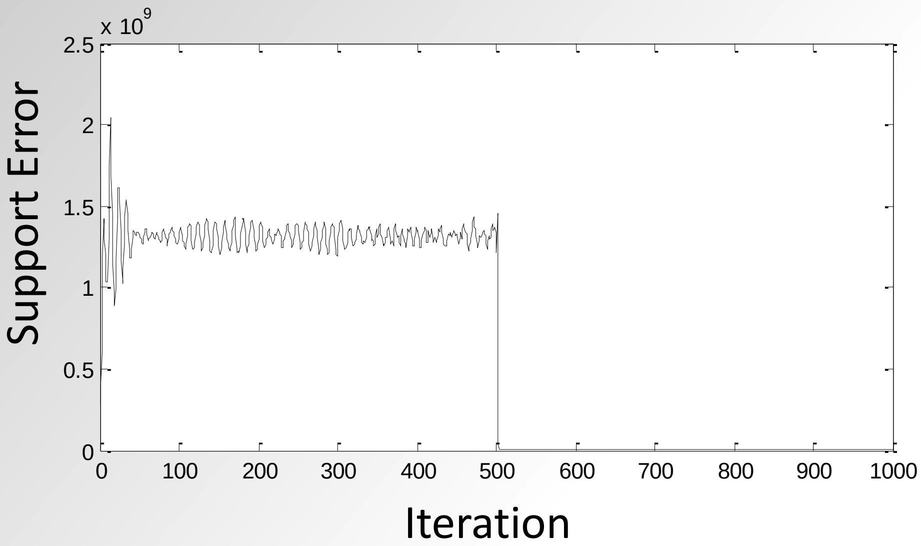

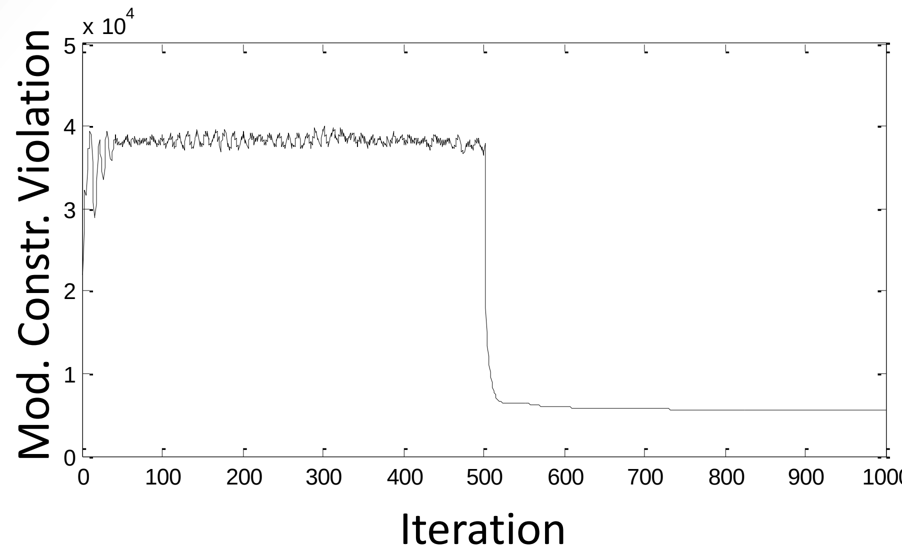

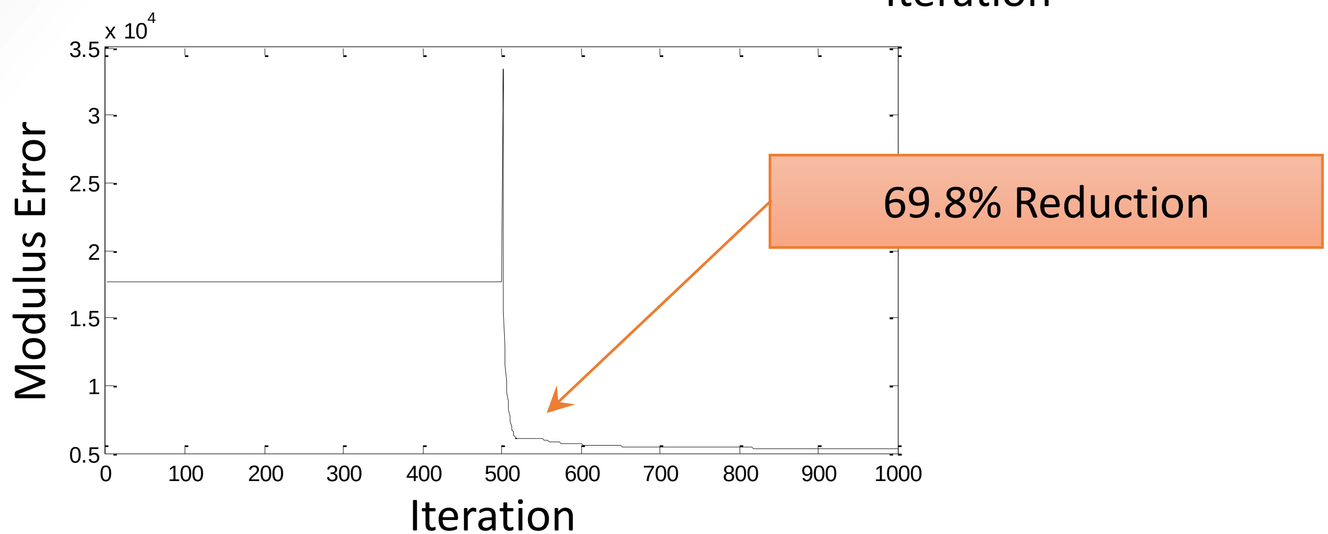

Conclusion

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- Phase retrieval methods have problem with convergence given low SNR data.
- Performed an analysis on how the noise affects the phase retrieval operators.
- Developed a new operator with a filtering rationale.
- Results show up to 75% noise is filtered in various examples with ~2x oversampling.

Analysis of the Effect of Noise

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- $G'(u,v) = \Delta\delta(u-a, v-b)$
- $g'(x,y) = \frac{\Delta}{MN}\exp\left(i2\pi\left(\frac{ax}{M}+\frac{by}{N}\right)\right)$

    Apply P_s

- $g(x,y) = \frac{\Delta}{MN}\begin{cases}1, & A \le m \le M-1-A \;\&\, B \le n \le N-1 \\ 0, & \text{otherwise}\end{cases}$
- $|G(u,v)| = \frac{\Delta}{MN}\frac{\sin\left(\frac{M-2A}{M}\pi u\right)}{\sin\left(\frac{\pi u}{M}\right)}\frac{\sin\left(\frac{N-2B}{N}\pi v\right)}{\sin\left(\frac{\pi v}{N}\right)}$

After one ER iteration, the Fourier modulus has less contribution from Δ.