

Low-CNR inverse synthetic aperture LADAR imaging demonstration with atmospheric turbulence

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Presented by Russell Trahan

Testbed ○○○○○○ CNR Derivation ○○○ Experimental Data ○○○○○ Conclusion ○

Goals:

- Demonstrate ISAL functionality in photon-starved conditions.
- Find a metric that can predict the success/failure of PGA based on the return signal strength.

Outline:

- Testbed hardware setup and data processing
	- Basic setup for low-CNR
	- Atmospheric turbulence synthesis
	- Data pipeline
- CNR
	- CNR definition for a single range-bin (including detector noise)
	- Various metrics based on CNR
	- Image quality metric to compare to metrics based on CNR
- Experimental Data
	- High CNR functionality tests
	- Low CNR imaging examples showing PGA failure at mean CNR=~0.25

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Testbed Hardware Setup and Data Processing

Testbed \bullet ○○○○ CNR Derivation ○○○ Experimental Data ○○○○○ Conclusion ○

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Transceiver / Target Layout

Testbed ●●○○○○ CNR Derivation ○○○ Experimental Data ○○○○○ Conclusion ○

Transceiver Assembly

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Transmitter Designs California Institute of Technology

- No atmospheric turbulence
	- Fiber termination and collimating lens
- Atmospheric turbulence

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- 1. Fiber Termination
- 2. Collimating Lens collimate light from fiber
- 3. Iris truncate Gaussian beam to FWHM
- 4. Focusing Lens focus collimated light through the phase wheel
- 5. Phase Wheel introduce phase error
- 6. Speckle Image focal point of focusing lens
- 7. Magnification Lens magnify the speckle image onto the target

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Testbed Overview

Testbed ●●●●●● CNR Derivation ○○○ Experimental Data ○○○○○ Conclusion ○

PGA Summary

Our best results came from starting the window at 75% of the cross range extent, allowing $\tilde{\varphi}$ to converge to nearly zero, then decreasing window size by 25%. Repeat until window is ~10 pixels in cross range.

Over-sampling in range or including range-bins with very low CNR shouldn't influence the phase increments. Simply includes noise in summation.

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CNR Derivation and Image Quality Metrics

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CNR Definition

• CNR is defined as **Estimate of carrier strength**
StdDev of estimate of carrier strength

• Measurement can be modeled as

- The carrier for a single range bin is $\eta_d \sqrt{\eta_h N_L N_s}$ exp $(i\varphi_s)$ mate of carrier strength

f estimate of carrier stre

deled as
 s) + *N*(0, σ_{SN}^2) + *N*(0, σ_{NEP}^2)

ange bin is $\eta_d \sqrt{\eta_h N_L N_S}$ exp(
 $\frac{2}{SN} \approx \eta_d \frac{N_L}{2}$

ance is $\sigma_{NEP}^2 = \frac{P_{NE}^2 \tau}{2h^2 v^2}$
- Shot noise variance is $\sigma_{SN}^2 \approx \eta_d \frac{N_L}{2}$ *L N*
- Detector NEP noise variance is $\sigma_{NEP}^2 = \frac{P_{NE}^2 \tau}{2h^2v^2}$ 2 \overline{a} \overline{a} $2\overline{a}$ $P_{NF}^2 \tau$ h^2v^2 τ and the state of τ and V a
- Model is used to estimate the carrier strength and its variance vation $\bullet \circ \circ$ Experimental Data $\circ \circ \circ \circ$ Conclusion \circ
 $\overline{1}$ O \cap

imate of carrier strength

of estimate of carrier strength

odeled as
 φ , $\frac{1}{1} \times (0, \sigma_{sw}^2) + N(0, \sigma_{sw}^2)$

range bin is $n_a \sqrt{n_b N_L N_s} \exp(i\$ er strength

carrier strength
 $(0,\sigma_{\scriptscriptstyle NEP}^2)\ \eta_{\scriptscriptstyle d}\sqrt{\eta_{\scriptscriptstyle h}N_{\scriptscriptstyle L}N_{\scriptscriptstyle S}}\exp(i\varphi_{\scriptscriptstyle s})$
 $\frac{P_{\scriptscriptstyle NEF}^2}{2h^2v^2}$

c strength and its variance

 d h L S s N N i exp 2 2 2 4 2 4 2 var 2 4 4 4 1 *L S S L S S S NEP NEP NEP d h d h d h L d h L d h L N N N CNR N N N N*

4/19/2016 SPIE 9846-14 ¹⁰ 2 2 exp exp 0, 0, *d h L S d h L S s SN NEP N N i N N i N N* ² 2 2 4 *N N N* 2 2 for 1 2 1 2 for 1 *S d h S S d h S d h d h d h S S d h N N CNR N N N N*

R. L. Lucke and L. J. Rickard, "Photon-limited synthetic-aperture imaging for planet surface studies planet surface studies," *Applied Optics,* vol. 41, no. 24, pp. 5084-5095, 2002.

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Quality Metric Selection Jet Propulsion Laboratory California Institute of Technology

Quality metrics based on pre-PGA data:

- # Photons in each range-bin Maximum, Mean, Sum, Sum of squares
- CNR of mean photons per range-bin
- CNR of each range-bin Maximum, Mean, Sum, Sum of squares
- Phase progression Variance of each range-bin Minimum, Mean, Sum, Sum of squares

Quality metric based on post-PGA result:

- Image Contrast-to-Noise Ratio
- $C = \frac{\text{mean}(\text{foreground}) \text{mean}(\text{background})}{\text{order}(\text{because})}$ stdev(background)
- Foreground region is determined based on a priori knowledge of the target.
- PGA performance cannot be assessed as C decreases past 1.

Primary Question:

What quality metric has a consistent value at the threshold where PGA doesn't work?

Immediate Question: What quality metric has a consistent value when the image contrast-to-noise ratio is 1?

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Contrast depends on Cross-Range Extent

Considering only a single range bin and a consistent CNR:

- The image contrast is inversely proportional to the number of cross-range bins populated by the target.
- Parseval's Theorem: $\sum_{n=0}^{N-1} |P_n|^2 = \frac{1}{N}$ $\frac{1}{N} \sum_{k=0}^{N-1} |p_k|^2$
- Sum of a single range-bin's magnitude over all pulses must equal the mean of the cross-range pixel values.
	- If a **single cross-range pixel** is filled by the target, **contrast will be high**.
	- If **several cross-range pixels** are filled by the target, **contrast will be low**.

*This idea is confirmed in the experimental data presented later.

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Imaging Examples ~2m Range to Target

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Sample Low CNR Result Jet Propulsion Laboratory California Institute of Technology

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JPL Logo on Spectralon Jet Propulsion Laboratory California Institute of Technology

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Satellite Image

Illumination Beam

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- Testbed build to perform ISAL studies
	- Short 2m or long 400m range-to-target
	- Synthesized atmospheric turbulence
	- High and very low CNR capabilities
- CNR Derivation
	- Rigorous derivation of CNR for a single range-bin
	- Quality metric for overall signal: "Mean CNR"
	- Quality metric for image: Contrast-to-Noise Ratio
- Experimental Results
	- Target cross-range extent decreases image contrast (for constant CNR)
	- PGA can work for simple images down to \sim 0.25 CNR
	- Atmospheric turbulence raises minimum CNR threshold to ~ 0.75

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Backup Slides

• Detector DC voltage determines local oscillator photon count:

$$
P_L = \frac{V_{DC}}{G_{DC}} \Rightarrow N_L = \frac{P_L \tau}{E_{Ph}}
$$

• The mean one sided PSD: $(j^{th}$ voltage measurement in the k^{th} pulse)

$$
P_u = \frac{2}{N_V N_P \delta f} \sum_{k=0}^{N_P - 1} \left| \sum_{j=0}^{N_V - 1} V_{j,k} \exp\left(-i2\pi \frac{j u}{N_V}\right) \right|^2, \quad u = [0, N_V - 1]
$$

• The number of photons in each range bin is given by:

- *d_d*: Detector area
- η_d : Detector efficiency
- *e*: Electron charge
G: Detector Gain
-
- η_h : Heterodyne efficiency
- *h*_{*h*}: # LO photons per pulse
 \vdots # measured photons
 \vdots # range bins N_t : #LO photons per pulse
- *n* : # measured photons
 N_R : # range bins
-
- : Detector area

Detector efficiency

Electron charge

Detector Gain

Heterodyne efficiency

: # LO photons per pulse

measured photons

: # range bins

: # signal photons per pulse

Plank's constant : Detector area

: Detector efficiency

Electron charge

Detector Gain

: Heterodyne efficiency

: # LO photons per pulse

measured photons

: # range bins

: # signal photons per pulse

Plank's constant

Pulse time *a* : Detector area
 a : Detector efficiency

: Electron charge

: Detector Gain
 h : Heterodyne efficiency
 *l*_{*L*} : # LO photons per pulse

: # measured photons
 k : # range bins
 *l*_s : # signal photons per p : Detector area

: Detector efficiency

Electron charge

: Detector Gain

: Heterodyne efficiency

: # LO photons per pulse

measured photons
 R_s : # range bins

: # signal photons per pulse

Plank's constant

Pulse ti : Detector area

: Detector efficiency

Electron charge

Detector Gain

: Heterodyne efficiency

: # LO photons per pulse

measured photons

: # range bins

: # signal photons per pulse

Plank's constant

Pulse time Detector area

Detector efficiency

Electron charge

Detector Gain

Heterodyne efficiency

: # LO photons per pulse

measured photons

: # range bins

: # signal photons per pulse

Plank's constant

Pulse time *A_a*: Detector area
 p_a: Detector efficiency
 e: Electron charge
 G: Detector Gain
 P_h: Heterodyne efficiency
 N_L: # LO photons per pulse
 n: # measured photons
 N_R: # range bins
 N_S: # signal N_s : # signal photons per pulse *n_a*: Detector efficiency
 e: Electron charge
 G: Detector Gain
 n_h: Heterodyne efficiency
 N_L: # LO photons per pulse
 n: # measured photons
 N_R: # range bins
 N_S: # signal photons per pulse
 h:
- t
-

• Total power at detector due to an E field is related to the mean field amplitude:

$$
P_d = \int_{A_d} \frac{1}{2} \left| E \exp(2\pi i f t + i\varphi) \right|^2 dA = \frac{hcN}{\lambda \tau} = \frac{1}{2} A_d \overline{E}^2
$$

$$
\overline{E}^2 = \frac{2hcN}{\lambda A_d \tau}
$$

• Detector output current due to single range element:

Substituting	CMR	DEFi>Q	W	DEFi>Q	W																	
• Total power at detector due to an E field is related to the mean field amplitude:	• Measurement quantity is expected number of signal photons plus $L_2 = \int_{x_0}^{x_0} \frac{1}{2} [E \exp(2\pi i \theta + i\phi)]^3 dA = \frac{hcN}{4\pi} = \frac{1}{2} A_z E^2$ \n	• IDENTIFY of the system of the system is																				
$E^2 - \frac{2hcN}{2\pi\sqrt{\pi}}$	$\pi - \frac{3hcN}{2\pi}$	$\pi - \frac{3hcN}{2\pi}$	$\pi - \frac{3hcN}{2\pi}$																			
• Determine	$I_2 - \frac{7hc}{2\pi}$	$\pi - \frac{3hc}{2\pi}$	$\pi - \frac{3hc}{2\pi}$	$\pi - \frac{3hc}{2\pi}$	$\pi - \frac{3hc}{2\pi}$	$\pi - \frac{3hc}{2\pi}$	$\pi - \frac{3hc}{2\pi}$	$\pi - \frac{3hc}{2\pi}$	$\pi - \frac{3hc}{2\pi}$	$\pi - \frac{3hc}{2\pi}$	$\pi - \frac{3hc}{2\pi}$	$\pi - \frac{3hc}{2\pi}$	$\pi - \frac{3hc}{2\pi}$	$\pi - \frac{3hc}{2\pi}$	$\pi - \frac{3hc}{2\pi}$	$\pi - \frac{3hc}{2\pi}$	$\pi - \frac{3hc}{2\pi}$	$\pi - \frac{3hc}{2\pi}$	$\pi - \frac{3hc}{2\pi}$	$\pi - \frac{3hc}{2\pi}$	$\pi - \frac{3hc}{2\pi}$	$\pi - \frac$

DFT of 2M samples of I_d at the carrier frequency:

$$
D(\Delta f) = \frac{\tau}{2M} \sum_{m=0}^{2M-1} 2\eta_d e \frac{\sqrt{\eta_h N_L N_S}}{\tau} \cos(2\pi \Delta f t_m + \varphi_s) \exp(-2\pi i \Delta f t_m)
$$
\n
$$
= \eta_d e \sqrt{\eta_h N_L N_S} \exp(i\varphi_s)
$$
\n
$$
\bullet \quad \text{CNR is defined as}
$$

• Measured quantity is expected number of signal photons plus complex noise: **Example 12**
 Latter Controller C Measured quantity is expected $\begin{array}{cc} A_d: \text{ I} \ \eta_a: \text{ I} \ \text{number of signal photons plus} \end{array}$
 $\begin{array}{ll} \eta_a: \text{ I} \ \eta_a: \text{ I} \ \text{complex noise:} & \alpha: \text{ I} \ \text{of} \text{ in } \mathbb{R} \end{array}$
 $\begin{array}{ll} \sqrt{\eta_a N_L \tilde{N}_s} \exp(i\varphi) = \eta_a \sqrt{\eta_a N_L N_s} \exp(i\varphi_s) + \eta_b: \text{ I} \ \eta_b: \text{ I} \ \text{$ Measured quantity is expected
 N_z: Detector area

number of signal photons plus
 $\frac{n}{q_z}$: Detector efficiency

complex noise:
 $\frac{n}{q_z\sqrt{\eta_s N_t N_s}} \exp(i\varphi) = \eta_s \sqrt{\eta_s N_t N_s} \exp(i\varphi_s) + \eta_s$: Hector Gain
 $\frac{n}{N_t}$: Hector Gain **Solution Subsetted**
 EXECUTE:
 EXECUT *deasured quantity is expected*
 d_n: Detect
 umber of signal photons plus
 *d*_n_d: Detect
 *d*_n $\sqrt{\eta_h N_L N_s}$ exp($i\varphi$)= $\eta_a \sqrt{\eta_h N_L N_s}$ exp($i\varphi$)+
 *n*_h: Heter
 N(0, σ_{ssv}^2)+*N*(0, $\sigma_{s\ell}^2$)
 n: # *Solutions* **Properties**
 Solutions Measured quantity is expected

number of signal photons plus
 η_s : Detector efficiency

complex noise:
 $\eta_s \sqrt{\eta_s N_L N_s} \exp(i\varphi) = \eta_s \sqrt{\eta_s N_L N_s} \exp(i\varphi_s) + \frac{\eta_s}{N_L}$: Hetchot Gain
 η_s Hetchot Gain
 η_s Hetchot Gain
 η_s Measured quantity is expected
 n_{λ} : Detector efficiency

complex noise:
 $\frac{n_{\lambda}}{\sqrt{n_{\lambda}N_{\lambda}N_{\lambda}}} \exp(i\varphi) = \frac{n_{\lambda}\sqrt{n_{\lambda}N_{\lambda}N_{\lambda}}\exp(i\varphi) + \frac{n_{\lambda}N_{\lambda}}{N_{\lambda}}\exp(i\varphi) + \frac{n_{\lambda}N_{\lambda}N_{\lambda}}{N_{\lambda}}\exp(i\varphi) + \frac{n_{\lambda}N_{\lambda}N_{\lambda}}{N_{\lambda}}$

$$
\eta_a \sqrt{\eta_h N_L \tilde{N}_s} \exp(i\varphi) = \eta_a \sqrt{\eta_h N_L N_s} \exp(i\varphi_s) +
$$
\n
$$
N_L: \text{ Heterodyne efficiency}
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\n
$$
N \left(0, \sigma_{SN}^2\right) + N \left(0, \sigma_{NEP}^2\right)
$$
\n
$$
N_R: \text{ H measured photons}
$$
\n
$$
N_R: \text{ H range bins}
$$

• Measurement has a variance $\frac{d}{dA}$ due to shot noise:

$$
\sigma_{SN}^2 = \eta_d \frac{N_L + N_S}{2} \approx \eta_d \frac{N_L}{2}, \ N_L >> N_s
$$

• Measurement has variance due to detector noise d quantity is expected

of signal photons plus
 n_a : Detector efficien

noise:
 $\exp(i\varphi) = \eta_a \sqrt{\eta_a N_L N_s} \exp(i\varphi_a) + \frac{N_b}{N_L}$: Heterodyne efficien
 $\exp(i\varphi) = \eta_a \sqrt{\eta_a N_L N_s} \exp(i\varphi_a) + \frac{N_b}{N_L}$: Heterodyne efficien
 $N(0, \sigma_{sw}^2$ and the properties of the strip of the asured quantity is expected

mber of signal photons plus
 η_i : Detector efficiency

mplex noise:
 $\eta_i \frac{N_h N_i \hat{N}_s}{\exp(i\varphi)} = \eta_s \sqrt{\eta_h N_c N_s} \exp(i\varphi_s) + \sum_{k=1}^{n} H_{k}$: Heredyne efficiency
 $N(0, \sigma_{sw}^2) + N(0, \sigma_{sw}^2)$
 η_i : d quantity is expected $\begin{array}{ll}\nA_i: \text{ Detection area} \\
\eta_i: \text{ Detection}} \\
\text{of signal photons plus} \\
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\text{reis} \\
\text{exp}(i\varphi) = \eta_x \sqrt{\eta_n N_L N_s} \exp(i\varphi_i) + \frac{\eta_n: \text{Heterodyne efficiency}}{N_L: \text{HCorodyne efficiency}} \\
N_L: \text{HCorlyon's per pulse} \\
N(0, \sigma_{sw}^2) + N(0, \sigma_{xge}^2) \\
\text{or} \\
\eta_k: \text{H range bins} \\
\text{Hessing} \\$ ity is expected
 $\begin{array}{ll}\n\text{ity is expected} & A_d: \text{Det} \\
\text{photons plus} & \eta_d: \text{Det} \\
\text{e}: \text{Elect} \\
G: \text{Det} \\
\sqrt{\eta_h N_L N_s} \exp(i\varphi_s) + \eta_h: \text{Heter} \\
0, \sigma_{SN}^2\end{array}$
 $\begin{array}{ll}\n\eta_h: \text{Heter} \\
\eta_h: \text{Heter} \\
N_L: \# \text{LO} \\
n: \# \text{me} \\
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\text{e}: \text$ **Sured quantity is expected**
 Note of signal photons plus
 Note of signal photons plus
 N i Detector of
 N i Detector
 N i Betector
 $N_L \overline{N_s}$ exp($i\varphi$) = $\eta_d \sqrt{\eta_s N_L N_s}$ exp($i\varphi_s$) +
 N_L : #LO photo $N(0$ **ntity is expected** $\frac{A_a}{\eta_a}$: Detector area
 nal photons plus $\frac{A_a}{\eta_a}$: Detector efficiency
 : Electron charge
 : Electron charge
 $\frac{A_a}{\eta_a}$: Heterodyne efficiency
 $N(0, \sigma_{sw}^2) + N(0, \sigma_{swp}^2)$
 $N(0, \sigma_{sw$ **y** is expected η_a : Detector area

ohotons plus
 η_a : Detector efficiency
 ϵ : Electron charge
 G : Detector Gain
 η_h : Heterodyne efficiency
 $\eta_h N_L N_s \exp(i\varphi_s) + N_L$: #LO photons per pulse
 η_s : # singal photons **and photons plus**
 $\begin{array}{ll}\n & \lambda_u: \text{ Detection area} \\
\text{and photons plus}\n & \lambda_u: \text{ Detection charge} \\
 & \text{Electron charge} \\
 & \text{Electron charge} \\
\text{S: \text{ Detection charge}} \\
 & \text{Fil}-\text{Hilb} \\
 & \text{Filb} \\
 N(0, \sigma_{\text{av}}^2) + N(0, \sigma_{\text{NEP}}^2) \\
 & \text{Filb} \\
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ignal photons plus

se:
 φ) = $\eta_a \sqrt{\eta_h N_L N_s}$ exp $(i\varphi_s)$ +
 η_a : He
 $N(0, \sigma_{sw}^2) + N(0, \sigma_{x\ell P}^2)$
 η_h : He
 $N(0, \sigma_{sw}^2) + N(0, \sigma_{x\ell P}^2)$

m: #1
 N_R : #1

mt has a variance

moise:
 $\kappa \$ is expected

otons plus
 $\begin{array}{ccc}\n a_i: & be \\ n_a: & be \\ \n b_i: & be \\ \n \hline\n \sqrt{k_1 N_s} \exp(i\varphi_s) + & \frac{1}{N_k}: & \frac{1}{N_k} \text{H} \\
 \hline\n \sqrt{k_1 N_s} \exp(i\varphi_s) + & \frac{1}{N_k}: & \frac{1}{N_k} \text{H} \\
 \hline\n \sqrt{k_1 N_s} \exp(i\varphi_s) + & \frac{1}{N_k}: & \frac{1}{N_k} \text{H} \\
 \hline\n \sqrt{k_1 N_s}$ Later of signal photons plus $\frac{N_t}{l}$. Detector area

lex noise:
 $\frac{N_s}{l} \exp(i\varphi) = \eta_s \sqrt{n_s N_t N_s} \exp(i\varphi) + \sum_{k=1}^{n} \frac{N_t}{l} \exp(i\varphi)$
 η_s : Heterodyna efficiency
 $\frac{N_s}{l} \exp(i\varphi) = \eta_s \sqrt{n_s N_t N_s} \exp(i\varphi) + \sum_{k=1}^{n} \frac{N_t}{l}$. **Example 12**
 Notation 12 *P P* **d** quantity is expected
 h_a: Detector area

f signal photons plus
 n_a : Detector efficience

c: Electron charge
 $\varphi(i\varphi) = \eta_a \sqrt{\eta_a N_L N_s} \exp(i\varphi_a) + \frac{\eta_a}{N_L}$: Heterodyne efficie
 $N(0, \sigma_{\text{max}}^2) + N(0, \sigma_{\text{max}}^2)$
 n_x quantity is expected A_a : Detector area

signal photons plus π_i : Detector efficiency

oise:
 $\left((i\varphi) = \eta_A/\eta_A N, N_s \exp(i\varphi_s) + N(0, \sigma_{\text{w}}^2) + N(0, \sigma_{\text{w}}^2)\right)$
 π_i : Heterodyne efficiency
 $N(0, \sigma_{\text{w}}^2) + N(0, \sigma_{\text{w}}$ staured quantity is expected

there of signal photons plus
 v_t : Elector efficiency

plex noise:
 $\sqrt{N_t N_s} \exp(i\varphi) = \eta_u \sqrt{\eta_s N_t N_s} \exp(i\varphi) + \frac{\sigma_t}{N_t}$: Heteroolyne efficiency
 $\sqrt{N_t N_s} \exp(i\varphi) = \eta_u \sqrt{\eta_s N_t N_s} \exp(i\varphi) + \frac{N_t}{N_t}$ and quantity is expected
 $\frac{A_i}{v_i}$: Detector area

ex noise:
 $\frac{1}{N_s} \exp(i\varphi) - \eta_s \sqrt{n_s N_s}$, $\exp(i\varphi_s) + \frac{1}{N_t}$: Heterodome efficiency
 $\frac{1}{N_s} \exp(i\varphi) - \eta_s \sqrt{n_s N_s}$, $\exp(i\varphi_s) + \frac{1}{N_t}$: $\frac{1}{n}$ to Denotons per pulse : Detector area

Detector efficiency

Electron charge

Detector Gain

Heterodyne efficiency

: # LO photons per pulse

measured photons

: # range bins

: # signal photons per pulse

Plank's constant

Pulse time

PLECTOR *a*: Detector area
 i: Detector efficiency

Electron charge

: Detector Gain
 i: Heterodyne efficiency
 *i*_z: # LO photons per pulse

: # measured photons
 *i*_{*s*}: # signal photons per pulse

Plank's constant

Pul *A_a*: Detector area
 n_a: Detector efficiency
 e: Electron charge
 G: Detector Gain
 n_h: Heterodyne efficiency
 N_k: # LO photons per pulse
 n: # measured photons
 N_R: # signal photons per pulse
 h

$$
\sigma_{NEP}^2 = \frac{1}{2} \left(\frac{P_{NE} \sqrt{\tau^{-1}} \lambda}{hc} \tau \right)^2 = \frac{P_{NE}^2 \lambda^2 \tau}{2h^2 c^2}
$$

• CNR is defined as Estimate of carrier strength StdDev of estimate of carrier strength

- *d*, : Detector area
- η_d : Detector efficiency
-
- *e*: Electron charge
G: Detector Gain
- η_h : Heterodyne efficiency
- N_t : #LO photons per pulse
- $(0, \sigma_{SN}^2)$ + *N* $(0, \sigma_{NEP}^2)$
n: # measured photons
N_R: # range bins
	-
- : Detector area

Detector efficiency

Electron charge

Detector Gain

Heterodyne efficiency

: # LO photons per pulse

measured photons

: # range bins

: # signal photons per pulse

Plank's constant : Detector area

: Detector efficiency

Electron charge

Detector Gain

: Heterodyne efficiency

: # LO photons per pulse

measured photons

: # range bins

: # signal photons per pulse

Plank's constant

Pulse time *a* : Detector area
 a : Detector efficiency

: Electron charge

: Detector Gain
 h : Heterodyne efficiency
 *l*_{*L*} : # LO photons per pulse

: # measured photons
 k : # range bins
 *l*_s : # signal photons per p : Detector area

: Detector efficiency

Electron charge

: Detector Gain

: Heterodyne efficiency

: # LO photons per pulse

measured photons
 R_s : # range bins

: # signal photons per pulse

Plank's constant

Pulse ti : Detector area

: Detector efficiency

Electron charge

Detector Gain

: Heterodyne efficiency

: # LO photons per pulse

measured photons

: # range bins

: # signal photons per pulse

Plank's constant

Pulse time Detector area

Detector efficiency

Electron charge

Detector Gain

Heterodyne efficiency

: # LO photons per pulse

measured photons

: # range bins

: # signal photons per pulse

Plank's constant

Pulse time *u*: Detector area
 u: Detector efficiency

Electron charge

: Electron charge

: Heterodyne efficiency
 u: # LO photons per pulse

: # measured photons
 *u*_R: # signal photons per pulse

: Plank's constant

: Pulse Detector area

Detector efficiency

Electron charge

Detector Gain

Heterodyne efficiency

: # LO photons per pulse

measured photons

: # range bins

signal photons per pulse

Plank's constant

Pulse time

tector *A_a*: Detector area
 p_a: Detector efficiency
 e: Electron charge
 G: Detector Gain
 P_h: Heterodyne efficiency
 N_L: # LO photons per pulse
 n: # measured photons
 N_R: # range bins
 N_S: # signal N_s : # signal photons per pulse *n_a*: Detector efficiency
 e: Electron charge
 G: Detector Gain
 n_h: Heterodyne efficiency
 N_L: # LO photons per pulse
 n: # measured photons
 N_R: # range bins
 N_S: # signal photons per pulse
 h:
	- t
	-

- CNR is defined as **Estimate of carrier strength**
StdDev of estimate of carrier strength
- Measurement gives number of detected photons $\widetilde{N}_{\mathcal{S}}.$

$$
\eta_d \sqrt{\eta_h N_L \tilde{N}_s} \exp(i\varphi) = \eta_d \sqrt{\eta_h N_L N_s} \exp(i\varphi_s) + N(0, \sigma_{SN}) + N(0, \sigma_{NEP})
$$

• Second moment gives estimate of \widetilde{N}_s

$$
\begin{aligned}\n\text{var}\Big(\eta_d\sqrt{\eta_h N_L \tilde{N}_s} \exp(i\varphi)\Big) &= \text{var}\Big(\eta_d\sqrt{\eta_h N_L N_s} \exp(i\varphi_s)\Big) + \text{var}\Big(N\big(0, \sigma_{\text{SN}}\big)\Big) + \text{var}\Big(N\big(0, \sigma_{\text{NEP}}\big)\Big) \\
\langle N_L \tilde{N}_S \rangle &= \langle N_L N_S \rangle + 2 \frac{\sigma_{\text{SN}}^2 + \sigma_{\text{NEP}}^2}{\eta_d^2 \eta_h}\n\end{aligned}
$$

• Fourth moment gives variance of \widetilde{N}_s

$$
\sum_{\text{Nndogy}} \text{CNN is defined as } \frac{\text{Estimate of carrier strength}}{\text{stable vector, } N_x, \overline{N}_x \exp(i\varphi) = \eta_x \sqrt{\eta_x N_x N_x} \exp(i\varphi) + N(0, \sigma_{xx}) + N(0, \sigma_{xx})} \quad \sigma_{xx}^2 \approx \eta_x \frac{N_x}{2} \quad \sigma_{x\alpha}^2 = \frac{P_{xx}^2}{2h^2 r^2}
$$
\n
$$
\frac{\text{second moment gives number of detected photons } \overline{N}_s}{\text{second moment gives estimate of } \overline{N}_s}
$$
\n
$$
\text{var}(\eta_x \sqrt{\eta_x N_x N_x} \exp(i\varphi)) = \text{var}(\eta_x \sqrt{\eta_x N_x N_x} \exp(i\varphi)) + N(0, \sigma_{xx}) + N(0, \sigma_{xx}) + \text{var}(\gamma(0, \sigma_{xx})) + \text{var}(\gamma(0, \sigma_{xx}))
$$
\n
$$
\frac{\partial \overline{X}_x}{\partial \overline{X}_y} = \frac{P_{xx}^2}{2h^2 r^2}
$$
\n
$$
\text{var}(\tilde{N}_x) = \langle N_x \rangle + \frac{2}{\sqrt{3}} \cdot \frac{1}{\sigma_{xx}^2}
$$
\n
$$
= \frac{2N_x}{\eta_x^2 \eta_x} + \frac{1}{\eta_x^2 \eta_x N_x} + \frac{4N_x \sigma_{x\alpha}^2}{\eta_x^2 \eta_x N_x} + \frac{4\sigma_{x\alpha}^2}{\eta_x^2 \eta_x N_x} + \frac{4\sigma_{x\alpha}^2}{\eta_x^2 \eta_x N_x}
$$
\n
$$
= \frac{2N_x}{\sqrt{\text{var}(N_x)}} = \frac{4N_x \sigma_{x\alpha}^2}{\sqrt{\text{var}(N_x \bar{N}_s)}} = \frac{N_x}{\sqrt{\frac{2N_x}{\eta_x \eta_x^2} + \frac{1}{\eta_x^2 \eta_x N_x} + \frac{4\sigma_{x\alpha}^2}{\eta_x^2 \eta_x N_x} + \frac{4\sigma_{x\alpha}^2}{\eta_x^2 \eta_x N_x} + \frac{4\sigma_{x\alpha}^2}{\eta_x^2 \eta_x N_x}}
$$
\n
$$
\text{CNR} = \frac{\langle N_x \tilde{N}_x \rangle}{\sqrt{\text{var}(N_x \bar{N}_x)}}
$$

•
$$
CNR = \frac{\langle N_L \tilde{N}_S \rangle}{\sqrt{\text{var}(N_L \tilde{N}_S)}} = \frac{N_S}{\sqrt{\frac{2N_S}{\eta_d \eta_h} + \frac{1}{\eta_d^2 \eta_h^2} + \frac{4N_S \sigma_{NEP}^2}{\eta_d^2 \eta_h N_L} + \frac{4\sigma_{NEP}^2}{\eta_d^4 \eta_h^2 N_L} + \frac{4\sigma_{NEP}^4}{\eta_d^4 \eta_h^2 N_L}}}
$$

d_d: Detector area

- η_{d} : Detector efficiency
- *e*: Electron charge
G: Detector Gain
-
- A_d : Detector area
 η_d : Detector efficie
 e : Electron charge
 G : Detector Gain
 η_h : Heterodyne effi
 N_L : #LO photons p
 n : # measured pho
 N_R : # range bins $\eta_{\scriptscriptstyle{k}}$: Heterodyne efficiency
- $\frac{\sigma_{L}}{2}$ $\sigma_{NEP}^{2} = \frac{I_{NE}t}{2h^{2}v^{2}}$ N_{L} : $\frac{N_L}{2}$ $\sigma_{NEP}^2 = \frac{P_{NE}^2 \tau}{2L^2 \tau^2}$ N_L : #LO photons p $\sigma_{NFP}^2 = \frac{P_{NE}^2 \tau}{I_{NE}^2}$ N_L : #LO photons per pulse *h*_{*h*}: # LO photons per pulse
 \vdots # measured photons
 \vdots # range bins N_t : #LO photons per pulse
	- h^2v^2
 n: # measured photons
 N_R : # range bins
		-
- : Detector area

Detector efficiency

Electron charge

Detector Gain

Heterodyne efficiency

: # LO photons per pulse

measured photons

: # range bins

: # signal photons per pulse

Plank's constant : Detector area

: Detector efficiency

Electron charge

Detector Gain

: Heterodyne efficiency

: # LO photons per pulse

measured photons

: # range bins

: # signal photons per pulse

Plank's constant

Pulse time *a* : Detector area
 a : Detector efficiency

: Electron charge

: Detector Gain
 h : Heterodyne efficiency
 *l*_{*L*} : # LO photons per pulse

: # measured photons
 k : # range bins
 *l*_s : # signal photons per p : Detector area

: Detector efficiency

Electron charge

: Detector Gain

: Heterodyne efficiency

: # LO photons per pulse

measured photons
 R_s : # range bins

: # signal photons per pulse

Plank's constant

Pulse ti : Detector area

: Detector efficiency

Electron charge

Detector Gain

: Heterodyne efficiency

: # LO photons per pulse

measured photons

: # range bins

: # signal photons per pulse

Plank's constant

Pulse time *A_a*: Detector area
 p_a: Detector efficiency
 e: Electron charge
 G: Detector Gain
 P_h: Heterodyne efficiency
 N_L: # LO photons per pulse
 n: # measured photons
 N_R: # range bins
 N_S: # signal N_c : # signal photons per pulse *n_a*: Detector efficiency
 e: Electron charge
 G: Detector Gain
 n_h: Heterodyne efficiency
 N_L: # LO photons per pulse
 n: # measured photons
 N_R: # range bins
 N_S: # signal photons per pulse
 h:
	- t
	-

Detector area

Detector efficiency

Electron charge

Detector Gain

Heterodyne efficiency

: # LO photons per pulse

measured photons

: # range bins

: # signal photons per pulse

Plank's constant

Pulse time

Since, f *u*: Detector area
 i: Electron charge
 \vdots Electron charge
 \vdots Electron charge
 \vdots # LO photons per pulse
 \vdots # measured photons
 \vdots # range bins
 \vdots # signal photons per pulse
 \vdots Plank's constant
 Detector area

Detector efficiency

Electron charge

Detector Gain

Heterodyne efficiency

: # LO photons per pulse

measured photons

: # range bins

signal photons per pulse

Plank's constant

Pulse time

.

.

.

. : Detector area

Detector efficiency

Electron charge

Detector Gain

Heterodyne efficiency

: # LO photons per pulse

measured photons

: # range bins

: # signal photons per pulse

Plank's constant

Pulse time

Pulse

$$
\frac{3}{2} \int \frac{1}{\sqrt{2\pi}} \int \frac{1}{\sqrt
$$

R. L. Lucke and L. J. Rickard, "Photon-limited synthetic-aperture imaging for planet surface studies planet surface studies," *Applied Optics,* vol. 41, no. 24, pp. 5084-5095, 2002.

2 N_L 2 F_{NF}

