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## ABSTRACT

In this paper we present a time-frequency approach for acoustic seabed classification. Work reported is based on sonar data collected by the Volume Search Sonar (VSS), one of the five sonar systems in the AN/AQS-20. The Volume Search Sonar is a beamformed multibeam sonar system with 27 fore and 27 aft beams, covering almost the entire water volume (from above horizontal, through vertical, back to above horizontal). The processing of a data set of measurement in shallow water is performed using the Fractional Fourier Transform algorithm in order to determine the impulse response of the sediment. The Fractional Fourier transform requires finding the optimum order of the transform that can be estimated based on the properties of the transmitted signal. Singular Value Decomposition and statistical properties of the Wigner and Choi-Williams distributions of the bottom impulse response are employed as features which are, in turn, used for classification. The Wigner distribution can be thought of as a signal energy distribution in joint time-frequency domain. Results of our study show that the proposed technique allows for accurate sediment classification of seafloor bottom data. Experimental results are shown and suggestions for future work are provided**.**

**Keywords:** Fractional Fourier transform, impulse response, seabed classification, time-frequency distributions, volume search sonar.

## 1. INTRODUCTION

In this paper we present a time-frequency approach for acoustic seabed classification. Work reported is based on sonar data collected by the Volume Search Sonar (VSS), one of the five sonar systems in the AN/AQS-20. The collection of data was performed in the Gulf of Mexico. The AQS-20 system is an underwater towed body containing a high resolution, side-looking, multibeam sonar system used for minehunting along the ocean bottom, as well as a forward looking sonar, and the volume search sonar which provided the data we use for our work. The system is illustrated in Figure 1.


Figure 1: AQS – 20 mine hunting sonar

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The VSS consists of two separate arrays: the transmit array and the receive array. The VSS is a beamformed multibeam sonar system with 54 beams arranged as 27 fore–aft beam pairs, covering almost the entire water volume (from above horizontal, through vertical, back to above horizontal) [1]. The VSS can be used in two modes: volume mode and SPD mode. The acoustic energy received by the VSS hydrophone array is pre-amplified and conditioned. Conditioning includes dynamic range compensation using time varying gain (TVG), bandshifting to IF, and band pass filtering. After conditioning, analog to digital (A/D) conversion is performed and signals are undersampled at 200 KHz. The beamforming function forms all beams and then quadrature demodulates the beam data to baseband. A hybrid time delay phase shift function is used to beamform by using a Hilbert transform after the element delays. The beam outputs are produced by shading (weighted sum) of array element data, delayed to compensate for cylindrical array geometry. Data from this sonar may be used for bathymetry computation, bottom classification, target detection, and water volume investigations[2].

The analysis presented here focuses on the bottom-return signals because we are interested in determining the impulse response of the ocean floor. The bottom-return signal is the convolution between the impulse response of the bottom floor and the transmitted sonar chirp signal. The objective of this paper is to investigate sediment classification based on singular value decomposition (SVD) of different time-frequency distributions applied to the impulse response of the seafloor. The time frequency distributions used in this paper are the Wigner distribution and the Choi Williams distribution. The impulse response has been obtained using two methods. The first method is standard deconvolution whereas the second method is based on the Fractional Fourier Transform (FrFT), a fundamental tool for optical information processing and signal processing. In recent years, interest in and use of time-frequency tools has increased and become more suitable for sonar and radar applications [3, 4, 5]. Both methods are presented in order to show that the proposed technique based on the Fractional Fourier Transform (FrFT) allows efficient sediment classification of seafloor bottom data compared to the conventional method based on deconvolution.

This paper starts with a presentation of the essential concepts and definitions related to the Fractional Fourier transform, time frequency distributions, and singular value decomposition. It then proceeds with the description of the proposed technique that improves the performance of unsupervised sediment classification. The conventional and the proposed methods are tested on real sonar data. The experimental results are shown and an their evaluation is carried out. The last section of the paper gives a summary of the presented work, conclusions, proposals for future work, and recommendations; references follow at the end.

## 2. THEORETICAL ASPECTS

### 2.1 Fractional Fourier Transform

The Fractional Fourier Transform (FrFT) is a generalization of the identity transform and the conventional Fourier transform (FT) into fractional domains. The fractional Fourier transform can be understood as a Fourier transform to the $a^{th}$ power, where $a$ is not required to be an integer. There are several ways to define the FrFT; the most direct and formal one is given by[6]:

$$\mathrm{f_a(u)} = \int_{-\infty}^{+\infty} \mathrm{K_a(u,u')f(u')du'} \tag{1}$$

where $K_a(u,u') \equiv A_\alpha \exp[i\pi(\cot\alpha \cdot u^2 - 2\csc\alpha \cdot u \cdot u' + \cot\alpha \cdot u'^2)]$ and $A_\alpha = \sqrt{1 - i\cot\alpha}$ , $\alpha = \dfrac{a\pi}{2}$ when $a \neq 2k$

$$K_a(u,u') \equiv \delta(u - u') \text{ when } a = 4k$$

$$K_a(u,u') \equiv \delta(u + u') \text{ when } a = 4k \pm 2$$

where $k$ is a integer and $A_\alpha$ is a constant term. The order of the transform is $a$. If we set $a$ = 1, which corresponds to $\alpha = \dfrac{\pi}{2}$ and $A_\alpha = 1$, the FrFT becomes the ordinary Fourier transform of $f(u)$:

$$f_1(u) = \int_{-\infty}^{+\infty} e^{-i2\pi uu'} f(u')du' \tag{2}$$

Due to periodic properties, the $a$ range can be restricted to (-2, 2] or [0, 4), respectively $\alpha \in (-\pi, \pi]$ or $\alpha \in [0, 2\pi)$. The fractional Fourier transform operator, $F^a$, satisfies important properties such as linearity, index additivity $F^{a_1}F^{a_2} = F^{a_1+a_2}$, commutativity $F^{a_1}F^{a_2} = F^{a_2}F^{a_1}$, and associativity $(F^{a_1}F^{a_2})F^{a_3} = F^{a_1}(F^{a_2}F^{a_3})$. In the operator notation, these identities follow[5]: $F^0=I$; $F^1=F$; $F^2=P$; $F^3=FP=PF$; $F^4=F^o=I$; and $F^{4k+a}=F^{4k'+a}$, where $I$ is an identity operator, $P$ is a parity operator, and k and k' are arbitrary integers. According to the above definition (1), the zero-order transform of a function is the same as the function itself *f(u)*, the first order transform is the Fourier transform of *f(u)*, and the $\pm 2^{nd}$ order transform is equal to *f(-u)*.

### 2.2 Time Frequency analysis

The generalized time-frequency representation can be expressed in term of the kernel, $\varphi(\theta,\tau)$, a two dimensional function[7]:

$$C(t,\varpi) = \frac{1}{4\pi^2}\iiint f^*(u-\tau/2)f(u+\tau/2)\varphi(\theta,\tau)e^{-j\theta t-j\tau\omega+ju\theta}dud\tau d\theta \qquad (3)$$

The kernel determines the properties of the distribution. Here are some distributions that can be obtained based on the value of the kernel :

1. Wigner distribution can be derived from the generalized time-frequency representation for $\varphi(\theta,\tau) = 1$:

$$W(t,\varpi) = \frac{1}{2\pi}\iint f^*(t-\tau/2)f(t+\tau/2)e^{-j\tau\varpi}d\tau \qquad (4)$$

2. Choi-Williams distribution obtained for $\varphi(\theta,\tau) = e^{-\theta^2\tau^2/\sigma}$:

$$C(t,\varpi) = \frac{1}{4\pi^{3/2}}\iint \frac{1}{\sqrt{\tau^2/\sigma}} f^*(u-\tau/2)f(u+\tau/2)e^{-\sigma(u-t)^2/\tau^2-j\tau\varpi}dud\tau \qquad (5)$$

3. Spectrogram obtained for $\varphi(\theta,\tau) = \int h^*(u-\tau/2)\, e^{-j\theta u}\, h(u+\tau/2)\, du$

$$C(t,\varpi) = \left|\frac{1}{\sqrt{2\pi}}\int e^{-j\varpi\tau} f(\tau)h(\tau-t)d\tau\right|^2 \qquad (6)$$

The Wigner distribution function is a time-frequency analysis tool and it can be used to illustrate the time-frequency properties of a signal; it can be interpreted as a function that indicates the distribution of the signal energy over the time-frequency space. The most significant properties of the Wigner distribution and the relationships between Wigner distribution and FrFT are stated in the following equations:

1. $\int W(t,\omega)\, d\omega = |f(t)|^2$ (7)

2. $\int W(t,\omega)dt = |F(\omega)|^2$ (8)

3. $\iint W(t,\omega)dt\, d\omega = \|f\|^2 = En[f]$ (9)

4. If $g(t) = h(t)*f(t)$,
   then $W_g(t,\omega) = \int W_h(t-\tau,\omega)\, W_f(\tau,\omega)d\tau$ (10)

5. If $g(t) = h(t)f(t)$,
   then $W_g(t,\omega) = \int W_h(t,\omega-\omega')W_f(t,\omega')d\omega'$ (11)

The Wigner distribution of the Fourier transform is the Wigner Distribution of the original function rotated clockwise by a right angle. The Wigner distribution is completely symmetric with respect to the time-frequency domain, it is always real but not always positive. The Wigner distribution exhibits advantages over the spectrogram (short-time Fourier transform): the conditional averages are exactly the instantaneous frequency and the group delay, whereas the spectrogram fails to achieve this result, no matter what window is chosen. The Wigner distribution is not a linear transformation, a fact that complicates the use of the Wigner distribution for time-frequency filtering.

One disadvantage of the Wigner distribution is that sometimes it indicates intensity in regions where one would expect zero values. These effects are due to cross terms but are minimized by choosing a different kernel. The kernel of the form $\varphi(\theta,\tau) = e^{-\theta^2\tau^2/\sigma}$, yields the Choi William distribution which, by appropriately choosing the parameter $\sigma$ minimizes the cross terms. When the Choi-Williams kernel is used, the marginals are satisfied and the distribution is real. In addition, if the σ parameter has a large value, the Choi-Williams distribution approaches the Wigner distribution, because the kernel approaches one. For small σ values it satisfies the reduced interference criterion.

### *2*.3 Singular value decomposition

A decomposition of joint time-frequency signal representation using the techniques of linear algebra, called singular value decomposition yields a qualitative signal analysis tool. The concept of decomposing a Wigner distribution in this manner was first presented by Marinovich and Eichman [8]. One motivation for such decomposition is noise reduction because when keeping only the first few terms most of the noise is lost; the other motivation for this decomposition is for the purpose of classification [7]. The basic idea in the latter case is that singular values contain unique characterization of the time-frequency structure of a distribution and may be used for classification. The set of representations of singular values is called the singular value spectrum of the signal. For the discrete Wigner distribution, the singular value decomposition (SVD) is given by [9]:

$$W = UDV^T = \sum_{i=1}^{N} \sigma_i u_i v_i^T, \tag{12}$$

$$\| W \|_F^2 = \sum_{i=1}^{N} \sigma_i^2$$

where T denotes transpose, $D = diag(\sigma_1, \sigma_2, \ldots \sigma_N)$ with singular values $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_N$, $U$ and $V$ are matrices that contain the singular vectors, and $\| W \|_F$ is the Frobenius norm matrix.

The properties of the Wigner distribution lead us to the conclusion that the volume under the surface that corresponds to a particular expansion term is equal to the signal energy contained in that term. Permutations of the rows (columns) or unitary transformation of $W$ lead to similarity transformations of $WW^T$ and $W^TW$. The singular values are invariant under these transformations and also invariant to time and/or frequency shifts in the signal. The number of non-zero spectrum coefficients equals the time-bandwidth product of the signal. Because singular values of the Wigner distribution encode certain invariant features of the signal, the set of singular values can be considered as the feature vectors that describe the signal and therefore be used in classification.

## 3. APPROACH

The proposed seafloor sediment classification method is based on singular value decomposition of the time-frequency distributions of the bottom impulse response. Our technique accurately determines the seafloor bottom characteristics and bottom type using the reverberation signal as the starting point.

The impulse response of the seafloor can be determined using the classical deconvolution method or using FrFT[10]. We have previously tested the techniques employed in this work for impulse response determination both on synthetic data and actual sonar data collected by the VSS[11,12]. The synthetic sonar return signal is generated by the convolution between the transmitted VSS chirp, with the specific parameters characterizing the system in question —such as a bandwidth of 10400 Hz, chirp duration of 4.32 ms[2,11]— and the Green function that has been utilized to simulate the synthetic impulse response of the seafloor.

Applying a classical deconvolution method to the return signal allows us to determine the bottom impulse response by taking the inverse Fourier transform of (13):

$$H(\varpi) = H_I(\varpi) + j \cdot H_Q(\varpi) \tag{13}$$

$$H_I(\varpi) = \frac{R_I(\varpi)P_I(\varpi) + R_Q(\varpi)P_Q(\varpi)}{P_I^2(\varpi) + P_Q^2(\varpi)} \tag{14}$$

$$H_Q(\varpi) = \frac{-R_I(\varpi)P_Q(\varpi) + R_Q(\varpi)P_I(\varpi)}{P_I^2(\varpi) + P_Q^2(\varpi)} \tag{15}$$

In the above equations $R(\varpi)$ and $P(\varpi)$ are the Fourier transforms of complex baseband received signal and of transmitted pulse, respectively. The subscripts I and Q denote, respectively, the real (in-phase) and imaginary (quadrature-phase) components of the complex signal.

The fractional Fourier transform requires finding the optimum order of the transform; the order can be estimated based on the properties of the chirp signal: the rate of change $\lambda$, sampling rate $f_s$, and the length of the data segment [3], $N$ :

$$a=(2/\pi)\tan^{-1}[f_s^2/(2N\lambda)] \tag{16}$$

The bottom impulse response is given by the magnitude of the Fractional Fourier transform for optimal order applied to the bottom return signal [10].

The amplitude and the shape of an acoustic signal reflected from the seafloor are determined mainly by the seabottom roughness, by the density difference between water and the seafloor, and by the reverberation within the substrate. Because accurate characterization of the distribution of the seafloor types is a very important tool in many commercial and military applications, we developed and implemented a sediment classification procedure which is based on the singular value decomposition of the Wigner and Choi Williams distributions of the obtained impulse response corresponding to each beam and each sediment class**.** Joint time-frequency representation of the signal offers the possibility of determining the time-frequency configuration of the signal as characteristic features for classification purposes. The sets of the singular values represent the desired feature vectors that describe the properties of the deconvolved signal and, therefore, the sediment. Because feature analysis involves dimensionality reduction, we consider the first two terms from the S.V. spectrum. A representation of the two singular values in two dimensional space will lead to unsupervised classification.

## 3. EXPERIMENTAL RESULTS

Our evaluation is based on actual data acquired by the VSS sonar. Two types of sediments are present in the area surveyed: sand and mud. The deconvolution method and the fractional Fourier transform method presented here have been applied to the same beams and pings before [10,11]. We used the same window of 256 samples for both methods. The optimum order of the fractional Fourier transform corresponds to the highest pulse compression and it was found to be 0.269 for this specific VSS chirp. To determine the order of the transform we used eq. (16) and we validated its value by examination of the chirp's Wigner distribution. In general, the Wigner distribution of the chirp function is found to be concentrated along the line giving the instantaneous frequency of the chirp. For the optimal order the Wigner distribution of the chirp becomes the delta function.

The amplitude of the fractional Fourier transform applied to the bottom return data for the optimum order represents the amplitude of the bottom's impulse response. Fig. 2 presents the normalized amplitude impulse response of sand corresponding to the nadir beam using the two methods.

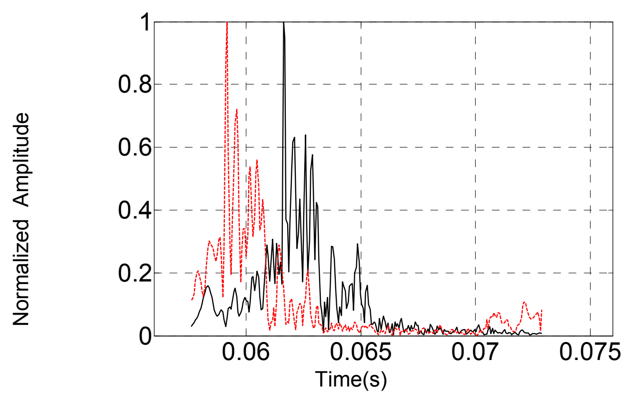


Figure 2: Normalized amplitude Impulse response of sand using standard deconvolution (red,dashed) and FrFT (black, solid) methods on the nadir beam.

The Wigner distribution is computed for each amplitude impulse response obtained via each method (standard deconvolution and fractional Fourier transform-based deconvolution) corresponding to each beam. Normalization is further performed and the singular values for each beam and each sediment class are computed as shown in our previous work [11].

The representation of the largest S.V. descriptors for each beam and each class has been obtained. A representation of the two singular values in two-dimensional space will lead to an unsupervised classification. The sediment classification is performed first using only 10 beams (corresponding to the central five beams from the fore and aft fans) and then 14 beams (the central seven beams from the fore and aft fans). Beam 27 (B27) and beam 28 (B28) correspond to nadir. The results are illustrated in Figs. 3 thru 6. Fig. 3 has been obtained applying the deconvolution method. Fig. 4 shows the results obtained when applying the FrFT method for the same data and the same window, for the 10 beams. Fig.5 uses the standard deconvolution method while Fig.6 has been achieved applying the FrFT method for the same data and the same window, but for the 14 central beams.

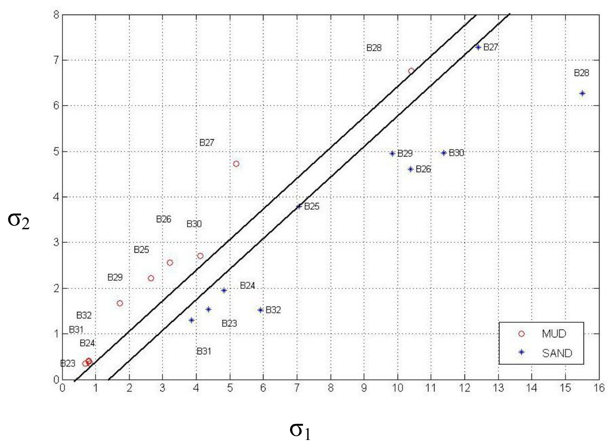


Figure 3: SV for Wigner distribution of impulse response using standard deconvolution method on 10 beams

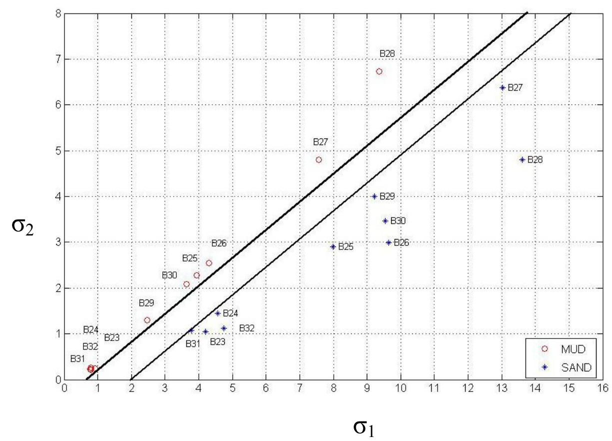


Figure 4: SV for Wigner distribution of impulse response using FrFT method on 10 beams

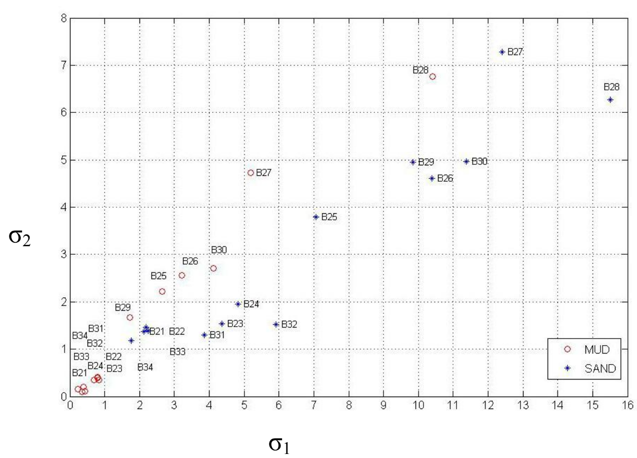


Figure 5: SV for Wigner distribution of impulse response using standard deconvolution method on 14 beams

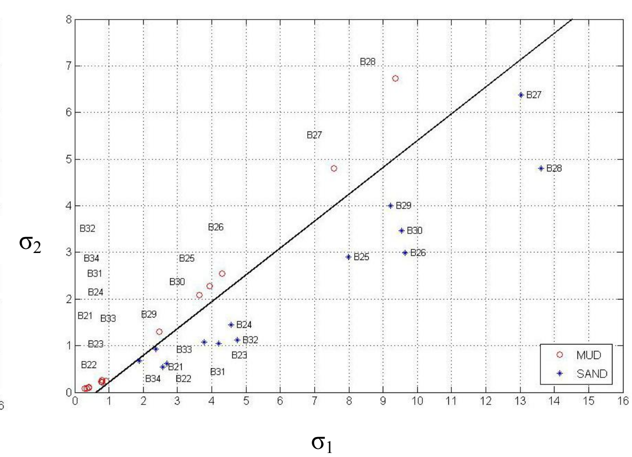


Figure 6: SV for Wigner distribution of impulse response using FrFT method on 14 beams

An improvement by approximately 50% can be observed for class separation from a margin width of 0.9 to 1.4 from Fig. 3 to Fig. 4, respectively. This improvement can be explained by the fact that parts of undesired signal components - such as noise - were eliminated when the impulse response was obtained using FrFT.

When 14 beams were analyzed, the return signal presented significant components due to the side lobes of the main return. These components negatively influence our classification. In the standard deconvolution method the two classes are linearly inseparable. The outer beams B21, B22, B31, B34 for sand and mud induce difficulties in separation. The mud class got a large undesired spread that requires a more complex evaluation. Using our FrFT method the two classes are barely linearly separable, but this method provides much better results compared to the standard deconvolution method.

In order to improve the classification performance, the Choi-Williams distribution was employed. These results are presented in Fig. 7 thru 10. One can observe an increase of the classification margin width for the 10 central beams when the Choi Williams distribution was applied to both the standard and the FrFT-based deconvolution methods as compard to the Wigner distribution. In the case of the 14 beams the two classes, Mud and Sand, became linearly separable with a larger margin width for both the standard deconvolution and the FrFT methods. This better classification performance is determined by the reduction of the cross interference terms when the Choi-Williams distribution is applied.

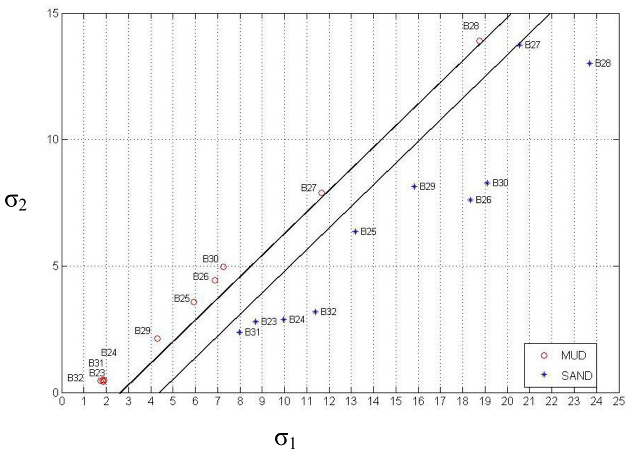


Figure 7: SV for Choi-Williams distribution of impulse response response using standard deconvolution on 10 beams.

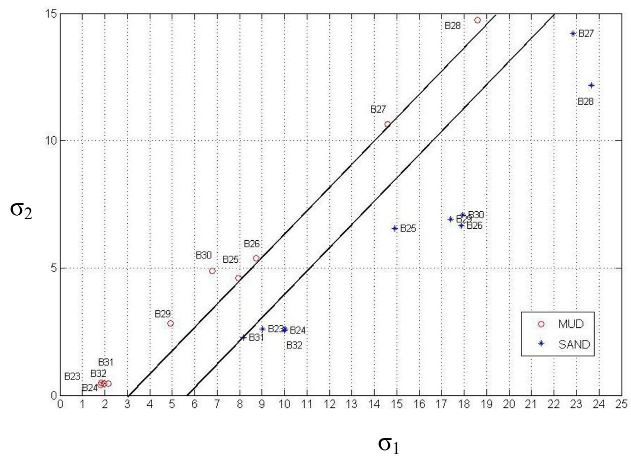


Figure 8: SV for Choi-Williams distribution of impulse impulse response using FrFT method on 10 beams.

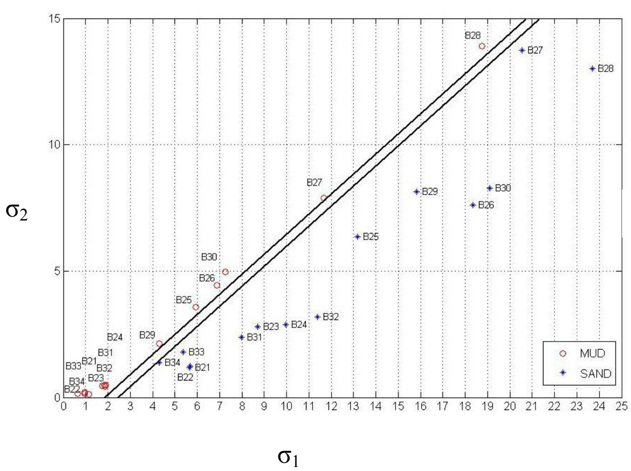


Figure 9: SV for Choi-Williams distribution of impulse response response using standard deconvolution on 14 beams.

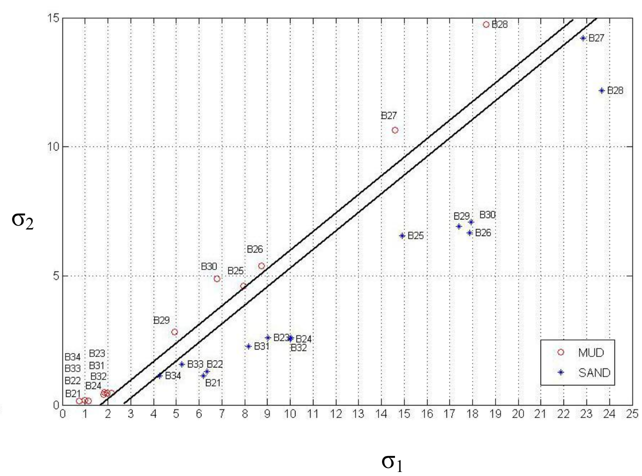


Figure 10: SV for Choi Williams distribution of impulse response using FrFT method on 14 beams.

Table 1: Margin width between classes for the techniques used.

| Number of beams used | Impulse response method | Margin width | |
|---|---|---|---|
| | | Wigner distribution | Choi Williams distribution |
| 10 | Standard deconvolution | 0.9 | 1.9 |
| 10 | FrFT deconvolution | 1.4 | 2.8 |
| 14 | Standard deconvolution | Not linearly separable | 0.5 |
| 14 | FrFT deconvolution | Barely linearly separable | 0.95 |

Table 1 illustrates the significant improvement of class separation that is achieved when the Choi-Williams distribution is used instead of the more popular Wigner distribution. Moreover, the best classification performance is achieved when fractional Fourier yransform and Choi-Williams distribution are employed together.

## 5. SUMMARY AND CONCLUSIONS

In this paper we presented an approach for processing of sonar signals with the ultimate goal of ocean bottom sediment classification. The approach is based on fractional Fourier transform (FrFT), a newly developed time-frequency analysis tool which has become attractive in signal processing. The best results on the final classification are achieved when singular value decomposition of the Choi-Williams distribution is applied to the impulse response obtained using deconvolution based on FrFT. Using Choi-Williams instead of Wigner improves both the classical deconvolution results and the Fr-FT-based results. The performance evaluation of our methods have been performed on data collected by the volume sonar system of the AQS-20 sonar system suite. The classification has proved to be very effective not only when the central beams, near nadir, were used, but also for a larger number of beams. The separation between the two classes, sand and mud, using our FrFT method together with the Choi-William distribution is approximately 60 % better than when classical deconvolution is used.

FrFT method with Choi Williams distribution showed a better classification performance than competing techniques due to its property of cross-terms interference reduction. Future work includes multi-class seabed classification using more features obtained from Choi Williams distribution applied to impulse response using FrFT.

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