DIGITAL SIGNAL PROCESSING 5, 167–175 (1995)

An Improved Method for Design of Recursive Digital Filters Using Optimization

Russell E. Trahan, Jr.,* Francis B. Grosz, Jr.,† and Sean Griffin†

*Department of Electrical Engineering, University of New Orleans, Louisiana 70148; and †Systems and Instrumentation Section Naval Research Laboratory, SSC Stennis Space Center, Mississippi 39529

Trahan, R. E., Jr., Grosz, F. B., Jr., and Griffin, S., An Improved Method for Design of Recursive Digital Filters Using Optimization, *Digital Signal Processing* **5** (1995) 167–175.

With the increased capability of today's personal computers (PC) and software packages, it often is advantageous to formulate engineering problems in such a way as to take full advantage of these resources. In this paper a new approach to recursive filter design is presented which takes advantage of a popular spreadsheet program and a simple formulation of an optimization problem. No code must be written other than the formula definitions in a spreadsheet. Explicit derivatives are not needed and full interaction with the optimization process is possible, including graphical information. Results are given for an eighth-order filter and these are compared to previously published designs.

1. INTRODUCTION

There are many methods available for the design of recursive digital filters. Some are based on the conversion of continuous time filters to discrete time using approximation techniques; others are based on direct design procedures. An excellent reference for techniques which can be used to design recursive filters satisfying prescribed specifications is Ref. [1]. In this book the author, Antoniou, provides various formulas and tables which can be used to translate filter specifications into filter designs. Also included is a section on design of recursive filters using optimization methods. Considerable interest in the application of optimization to the design of recursive digital filters has been shown over the past two decades [2,3]. With the tremendous improvements in optimization techniques it is now possible to design digital filters which have arbitrary amplitude or phase responses.

There are commercially available spreadsheet programs which have a built-in optimization capability. In one, the optimization techniques are selectable; either a quasi-Newton method or a conjugate–gradient method can be chosen. Furthermore, constraints on the optimization problem can be specified. All of these capabilities are available without the explicit use of derivatives. All that is necessary is the formulation of a cost function, constraint functions, if any, and the specification of variables. There are other commercially available software packages which run on personal computers (PCs) which also include optimization capabilities. These may also be used for the design of digital filters; the spreadsheet approach, however, will be described here. In terms of design time, the use of a commercially available software package decreases the time normally used for writing custom code. Since a spreadsheet program or some other mathematical computation package is usually available on PC work stations used by design engineers it makes sense to pose a problem in such a way to maximize the use of these standard tools.

In Section 2 of this paper the problem of recursive digital filter design using optimization is presented. A cost function is developed which is based on the formulation in [1]. The results are a least squares approach and a minimax criterion. In Section 3 a brief discussion of the existing methods used to solve the optimization problems will be presented. In Section 4 a new method will be presented. The new method is based on the work presented in [1] but by introducing a modification to the cost function an improved criterion is obtained. Results using this new formulation

1051-2004/95 $12.00

are presented in Section 5. Conclusions and suggestions for further work are presented in Section 6.

2. PROBLEM FORMULATION

The problem of recursive digital filter design is posed as follows:
Given an arbitrary magnitude and/or phase response as a function of frequency, find a stable rational function H(z) which most closely matches the desired response.

The desired magnitude and phase responses are defined by the particular specifications for the system to be designed. For illustrative purposes in this paper only the magnitude response will be used; the phase response, however, can be added to the design criterion. See [10] for a discussion of a technique for including phase in the criterion. Let the desired magnitude response be defined as

$$H_d(\omega) \qquad \omega \in [\Omega_l, \Omega_u], \tag{1}$$

where $[\Omega_l, \Omega_u]$ is the interval of frequencies of interest. Now define an Nth order digital filter transfer function by

$$H(z) = k_0 \prod_{i=1}^{I} \frac{a_{0i} + a_{1i}z + z^2}{b_{0i} + b_{1i}z + z^2}, \tag{2}$$

where $N = 2I$, and k_0, a_{0i}, a_{1i}, b_{0i}, b_{1i}, $i = 1, \ldots, I$, are the digital filter coefficients. The frequency response of this transfer function is obtained by setting $z = e^{j\omega T}$ in (2) to obtain

$$H(e^{j\omega T}) = k_0 \prod_{i=1}^{I} \frac{a_{0i} + a_{1i}e^{j\omega T} + e^{j\omega T2}}{b_{0i} + b_{1i}e^{j\omega T} + e^{j\omega T2}} \tag{3}$$

where T is the sample period in seconds. The magnitude of the transfer function, defined as $H_m(\omega)$, is obtained by solving for the magnitude of the numerator and denominator terms separately as

$$H_m(\omega) = |H(e^{j\omega T})| = \prod_{i=1}^{I} \frac{N_i(\omega)^{1/2}}{D_i(\omega)^{1/2}}, \tag{4}$$

where

$$N_i(\omega) = [1 + a_{0i}^2 + a_{1i}^2 + 2a_{1i}(1 + a_{0i})\cos \omega T + 2a_{0i}\cos 2\omega T] \tag{5}$$

and

$$D_i(\omega) = [1 + b_{0i}^2 + b_{1i}^2 + 2b_{1i}(1 + b_{0i})\cos \omega T + 2b_{0i}\cos 2\omega T]. \tag{6}$$

The magnitude of the transfer function in (4) can be compared to the desired magnitude in (1) and an error function can be defined as

$$e(\omega) = H_m(\omega) - H_d(\omega). \tag{7}$$

This error function can be evaluated at discrete frequencies and the individual errors summed in some manner to form an objective function for an optimization problem. A general form of a scalar objective function for this problem can be formulated in terms of an L_p norm by defining

$$E_p = [\sum_{l=1}^{L} W(\omega_l)\,|e(\omega_l)|^p]^{1/p}, \tag{8}$$

where $\{\omega_l \in [\Omega_l, \Omega_u] \,|\, l = 1, \cdots L\}$ is a set of discrete frequencies. This set of mesh points must be sufficiently dense to avoid spikes from occurring in the response between the points. The weighting function $W(\omega_l) \geq 0$ is chosen to emphasize certain frequencies. The choice of p in (8) changes the type of optimization. For $p = 2$ the problem becomes one of ordinary weighted least-squares.

Let the maximum error over the set of mesh points be defined by

$$e_{\max} = \max_{1 \leq l \leq L} |e(\omega_l)| \neq 0. \tag{9}$$

Then the limit as $p \rightarrow \infty$ gives

$$E_\infty = \lim_{p \to \infty} E_p = e_{\max} \lim_{p \to \infty} \left\{ \sum_{l=1}^{L} W(\omega_l) \left[\frac{|e(\omega_l)|}{e_{\max}} \right]^p \right\}^{1/p} = e_{\max}. \tag{10}$$

Therefore, with $p = \infty$ minimizing the objective function is a minimax problem.

In the problem statement above, the rational function $H(z)$ which is to be found must also be stable. Of course, this requirement is necessary in order to ensure a stable digital filter. There are two ways in which stability of the resultant filter is ensured. The first method is to design the filter without regarding the stability issue. If the transfer function which results from the optimization procedure is unstable (i.e., poles outside the unit circle), then the unstable poles are mapped into their reciprocals and the gain is adjusted to give the same DC filter gain. See Ref. [1] for a discussion of this stabilization technique. One disadvantage of this method is that phase distortion is introduced if the unstable poles are replaced by their reciprocals.

Another more direct approach to ensure stability is to use constraints in the optimization procedure. Since the denominator of $H(z)$ consists of second-order terms it is possible to constrain the roots of each term to be inside the unit circle. Given a second-order polynomial

$$b(z) = z^2 + b_1 z + b_0, \tag{11}$$

the roots are inside the unit circle if the following equations obtained using the Jury test are satisfied [4]:

$$\begin{aligned} b_1 &< b_0 + 1 \\ -b_1 &< b_0 + 1 \\ -1 &< b_0 < 1. \end{aligned} \tag{12}$$

These inequalities can be rewritten in standard inequality constraint form

$$\begin{aligned} 0 &< b_0 + 1 - b_1 \\ 0 &< b_0 + 1 + b_1 \\ 0 &< b_0 + 1 \\ 0 &< 1 - b_0. \end{aligned} \tag{13}$$

These constraints can be written for each second-order denominator term and the resultant constrained optimization problem is one in which the objective function is minimized subject to these constraints.

3. EXISTING METHODS

In [1] Antoniou defines the design of a recursive digital filter as a solution of the optimization problem

$$\min_{\mathbf{x}} \Psi(\mathbf{x}), \tag{14}$$

where the objective function $\Psi(\mathbf{x})$ is

$$\Psi(\mathbf{x}) = E_p \tag{15}$$

for $p = 2$ or ∞, and

$$\mathbf{x} = [a_{01}, a_{11}, b_{01}, b_{11}, \ldots, b_{1J}, k_0]^T \tag{16}$$

is the vector of unknown parameters to be optimally chosen. (Antoniou does not include a weighting function in the definition of E_p.) The optimization methods proposed by Antoniou are based on the popular quasi-Newton algorithms [5]. In all of the algorithms the derivative of the objective function must be computed; all are iterative and line searches must be made in each iteration.

In order to obtain a minimax solution Antoniou proposes a basic Least pth algorithm in which successive optimization problems are solved beginning with $p = 2$. Once a solution (actually a near solution) is obtained, the value of p is multiplied by 2 and the problem is solved again using the previous solution as an initial value. In this manner as the value of p is increased the minimax solution is approached [6].

Improved minimax algorithms are also described in [1]. These improved algorithms are based on the work of Charalambous [7,8] and use a modified objective function. Minimax multipliers are introduced and successive minimizations are performed with multiplier updating formulas used between each minimization.

All of the algorithms described in [1] for the design of recursive digital filters require significant coding efforts if attempted from scratch. Published subroutines such as those contained in [9] can be used, but there is still a significant effort required for the main program code. For an engineer, it is far better to use existing easy-to-use commercial software packages than to write custom code. The next section illustrates that this is possible for the design of recursive digital filters.

4. DESCRIPTION OF NEW TECHNIQUE

In the field of analog filter design it is customary to describe the gain over frequency ranges in units of deciBels (dB). By using 20 times the logarithm of the magnitude of the gain a large dynamic range can be accommodated. It will be shown here that using the error expressed in dB provides an improved objective function.

The problem of matching a rational function to a given frequency response in order to obtain a digital filter is very similar to the problem of identifying a system transfer function from a Bode plot for the system. Recently Sidman *et al.* presented results in which frequency response data were used to identify continuous time system transfer functions using logarithmic data [10].

In order to apply the ideas in [10] to the problem of digital filter design, the error function must be redefined. Let the error between the computed and desired responses be defined by

$$\bar{e}(\omega) = 20 \log_{10} H_m(\omega) - 20 \log_{10} H_d(\omega). \quad (17)$$

Note that this error criterion is equivalent to finding the ratio of the two gains, i.e.,

$$\bar{e}(\omega) = 20 \log_{10} \frac{H_m(\omega)}{H_d(\omega)}. \quad (18)$$

The use of a ratio provides an advantage when the desired gain $H_d(\omega)$ is very small. The log of a ratio of two small numbers can be minimized much more easily than the absolute value of the difference between the numbers. This advantage is demonstrated in [10] where rational functions are matched to frequency response data over a wide dynamic range.

Using (4) in (17) yields

$$\bar{e}(\omega) = 20 \log_{10} \prod_{i=1}^{I} \frac{N_i(\omega)^{1/2}}{D_i(\omega)^{1/2}} - 20 \log_{10} H_d(\omega), \quad (19)$$

and this can be further simplified to

$$\bar{e}(\omega) = \sum_{i=1}^{I} [10 \log_{10} N_i(\omega) - 10 \log_{10} D_i(\omega)] - 20 \log_{10} H_d(\omega). \quad (20)$$

Note that each term being summed in (20) has units of dB. The objective function is a weighted sum of all of the errors at the discrete frequencies. Using (20) to evaluate $\bar{e}(\omega)$ at the mesh points of the frequency range of interest and substituting into (8) yields

$$E_p = [\sum_{l=1}^{L} W(\omega_l) | \sum_{i=1}^{I} [10 \log_{10} N_i(\omega_l) - 10 \log_{10} D_i(\omega_l)] - 20 \log_{10} H_d(\omega_l) |^p]^{1/p}. \quad (21)$$

This is the objective function which can easily be entered into a spreadsheet and optimized.

5. COMPUTATIONAL RESULTS

The results obtained using the algorithm described above will be presented via an example taken from [1] (Example 14.4). The problem is to design an eighth order digital filter with a sample rate of $\omega_s = 2$ rad/s and a desired magnitude frequency response as shown in Figs. 1a and 1b. This response was actually specified at 21 evenly spaced discrete points beginning at 0.00 rad/s and ending at 1.00 rad/s.

The objective function for the design, which is defined by (21), can be entered into a spreadsheet with the columns containing each term evaluated at a single frequency. Graphically, the spreadsheet can be illustrated as

ω	α_1	β_1	α_2	β_2	α_3	β_3	α_4	β_4	Σ	M	E
0.0	α_{11}	β_{11}	α_{21}	β_{21}	α_{31}	β_{31}	α_{41}	β_{41}	Σ_1	M_1	E_1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
ω_l	α_{1l}	β_{1l}	α_{2l}	β_{2l}	α_{3l}	β_{3l}	α_{4l}	β_{4l}	Σ_l	M_l	E_l
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
											$\hat{\Sigma}$

The individual terms in the array represent the formulas entered in the spreadsheet. The first column is the frequency in rad/s incremented in 0.05 rad/s steps. The α_k and β_k, $k = 1, \ldots, 4$ columns represent the zero and pole terms, respectively, and the column entries are defined as

$$\alpha_{il} = 10 \log_{10}[1 + a_{0i}^2 + a_{1i}^2 + 2a_{1i}(1 + a_{0i})\cos \omega_l T + 2a_{0i}\cos 2\omega_l T] \quad (22)$$

$$\beta_{il} = -10 \log_{10}[1 + b_{0i}^2 + b_{1i}^2 + 2b_{1i}(1 + b_{0i})\cos \omega_l T + 2b_{0i}\cos 2\omega_l T]. \quad (23)$$

The Σ_l terms are given by

$$\Sigma_l = \sum_{i=1}^{4} (\alpha_{il} + \beta_{il}) + 20 \log_{10} k_0, \quad (24)$$

and for $p = 2$ the weighted error terms, E_l are

$$E_l = W(\omega_l)(\Sigma_l - M_l)^2, \quad (25)$$

where M_l is the desired magnitude in dB of the filter gain at the lth frequency. The weighting function was determined experimentally. The function $W(\omega_l) = \omega_l^{-0.8}$ (with $\omega_1 = 0.1$ substituted for $\omega_1 = 0$ to avoid dividing by zero) worked well to ensure a good fit at lower frequencies. Finally, the objective function (sum of all individual errors) is given by

$$\hat{\Sigma} = \sum_{l=1}^{L} E_l, \quad (26)$$

where $L = 21$. The square root of the sum of errors, which is required in (8), is not taken since minimizing the square is equivalent to minimizing the square root. The objective function is defined as a single cell for the spreadsheet optimization procedure.

It should be emphasized that the weighting func-

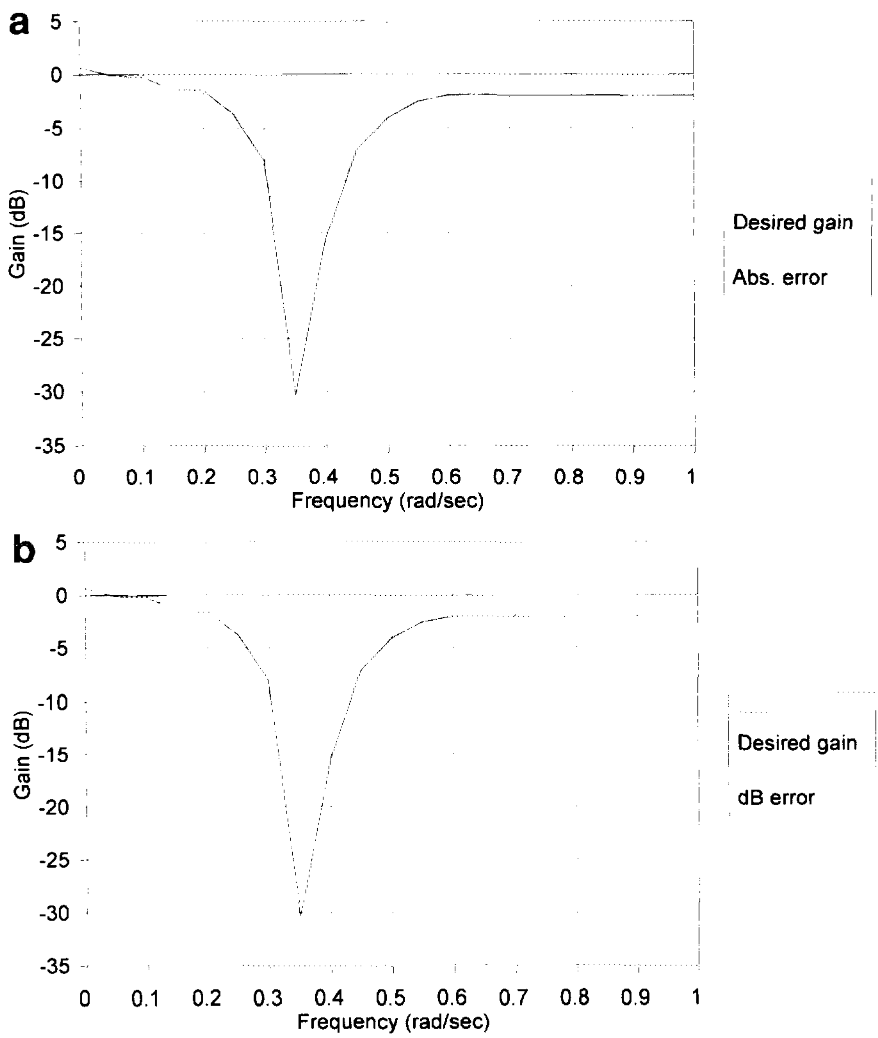

FIG. 1. Desired magnitude response for eighth-order digital filter and filter response for (a) Least-Squares design using absolute error, $e(\omega)$, and (b) least-squares design using dB error, $\bar{e}(\omega)$.

tion can be formulated in many different ways. For example, in experimenting with various weighting functions, one that was implemented was simply the multiplication of the midband frequency terms by 10. Thus, the optimization procedure concentrated more heavily on decreasing the midband errors. The power of the spreadsheet approach is that it is very simple to modify the error terms, run the optimization procedure, and look at the graphs of the results, all with the touch of a few keys or clicks of a mouse.

Stability of the resultant transfer function is ensured if each second-order denominator term is constrained to have roots inside the unit circle as discussed above. Thus, the constraints

TABLE 1

Initial Values Used for Example Digital Filter Design

a_{01}	1.00	b_{01}	0.75
a_{11}	1.00	b_{11}	1.00
a_{02}	1.00	b_{02}	0.75
a_{12}	1.00	b_{12}	1.00
a_{03}	1.00	b_{03}	0.75
a_{13}	−1.00	b_{13}	−1.00
a_{04}	1.00	b_{04}	0.75
a_{14}	−1.00	b_{14}	−1.00
k_0	1.00		

TABLE 2

Computation Times and Final Objective Function Values for Least-Squares

Case	Objective function	$\hat{\Sigma}^{1/2}$	$\max_l \lvert e(\omega_l)\rvert$	Run time (s)
(i)	$\hat{\Sigma} = \sum_{l=1}^{L} e(\omega_l)^2$	0.097828	0.062803	200+
(ii)	$\hat{\Sigma} = \sum_{l=1}^{L} \omega^{-0.8} e(\omega_l)^2$	0.176195	0.047066	300+
(iii)	$\hat{\Sigma} = \sum_{l=1}^{L} \bar{e}(\omega_l)^2$	0.955443	0.069838	200+
(iv)	$\hat{\Sigma} = \sum_{l=1}^{L} \omega^{-0.8} \bar{e}(\omega_l)^2$	1.615664	0.056329	400+

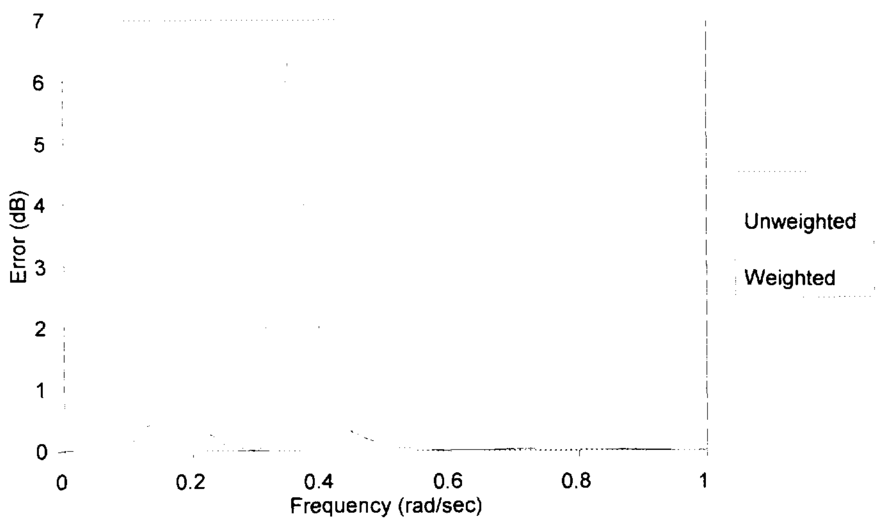

FIG. 2. Error response for cases (i) and (ii) of least-squares design (absolute error used).

$$0 < b_{0i} + 1 - b_{1i}$$

$$0 < b_{0i} + 1 + b_{1i}$$

$$0 < b_{0i} + 1$$

$$0 < 1 - b_{0i}$$

are used, where $i = 1, 2, \ldots, 4$. Each constraint equation is contained in a single cell in the spreadsheet.

The parameters to be optimized and the initial values are shown in Table 1; the parameters are simply defined as a block in the spreadsheet. The initial values are the same as those given in [1] (Example 14.4). As pointed out in [1] the final solution to any optimization problem is dependent on the choice of an initial solution. If the initial values are close to the optimal solution then the amount of computation may be low and the accuracy high. Generally, a good initial guess can be obtained from some nonoptimal procedure which results in a filter having the same general shape as the desired filter.

Using the initial values given in Table 1 four different least squares optimizations were attempted using the quasi-Newton algorithm contained in one available spreadsheet program. The four cases are

(i) error = $e(\omega_l)$ (absolute error) – no weighting,
(ii) error = $e(\omega_l)$ (absolute error) – weighting, $W(\omega_l) = \omega_l^{0.8}$,
(iii) error = $\bar{e}(\omega_l)$ (dB error) – no weighting,
(iv) error = $\bar{e}(\omega_l)$ (dB error) – weighting, $W(\omega_l) = \omega_l^{0.8}$.

The results for the four cases are shown in Table 2.

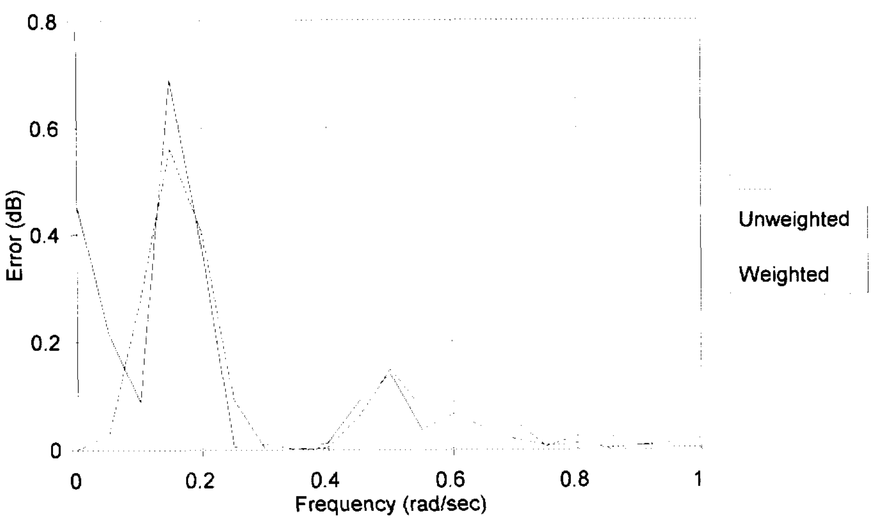

FIG. 3. Error response for cases (iii) and (iv) of least-squares design (error in dB used).

TABLE 3

Computation Times and Final Objective Function Values for Least pth Method

p	$\hat{\Sigma}$	e_{max}	Run time (s)
	Unweighted		
$p = 2$	0.961251	0.070555	100+
$p = 4$	0.682587	0.057973	100+
$p = 8$	0.582573	0.057364	100+
$p = 16$	0.533772	0.058009	100+
$p = 32$	0.509454	0.058267	100+
$p = 64$	0.498751	0.058547	<100
$p = 128$	0.490013	0.058298	100+
	Weighted $W(\omega_l) = \omega_l^{0.8}$		
$p = 2$	3.070302	0.090781	100+
$p = 4$	0.800280	0.042441	300+
$p = 8$	0.541371	0.039492	<100
$p = 16$	0.447329	0.039834	<100
$p = 32$	0.403249	0.040080	<100
$p = 64$	0.386108	0.040097	<100
$p = 128$	0.379032	0.040096	<100

The column headed by $\hat{\Sigma}^{1/2}$ is the square root of the final value of the objective function. The next column contains the maximum absolute error in the gain, and the last column contains the run times on an 80486 DX2/50 computer with 16 megabytes of RAM. The optimization program was run in increments of 100 s. The procedure was automatically terminated when the objective function began changing too slowly. The plus sign on the run times in Table 2 signifies that the last full 100 s increment of run time was not completed.

The frequency responses for the resultant design from cases (ii) and (iv) are shown in Figs. 1a and 1b, respectively. Note that the absolute error in case (ii) is very small but the frequency response is off by over 6 dB at the notch. This error is attributable to the fact that the error at the notch frequency, though small in absolute value, is large relative to the small desired gain. In case (iv) the error in dB is used and the resultant design provides a very close match across the spectrum, including the notch frequency.

The errors as a function of frequency for cases (i) and (ii) are shown in Fig. 2. Note that the error is shown in dB. Case (i), since it does not include a weighting function, does not match the gain as well at low frequencies. By using the weighting function to emphasize the low frequencies in case (ii) a better match is made at these frequencies at the expense of a larger error at the notch frequency. The errors for cases (iii) and (iv) are shown in Fig. 3. The errors are quite small in both cases, including the errors at the notch frequency. The objective function with the weighting function gives a closer match at the lower frequencies.

The least pth objective function was implemented in a spreadsheet similar to that of the least-squares design described above. The objective function was defined as

$$\hat{\Sigma} = \hat{e}_{max} \left[\sum_{l=1}^{L} W(\omega_l) \left(\frac{\bar{e}(\omega_l)}{\bar{e}_{max}} \right)^p \right]^{1/p}, \tag{27}$$

where

$$\hat{e}_{max} = \max_{1 \le l \le L} |\bar{e}(\omega_l)|. \tag{28}$$

The results with $p = 2, 4, 8, \ldots, 128$ are shown in Table 3. Two runs were made, one with $W(\omega_l) \equiv 1$ and one with $W(\omega_l) = \omega_l^{0.8}$. The frequency responses of the final designs for the unweighted and weighted cases are shown in Figs. 4a and 4b, respectively. The actual errors are shown in Fig. 5. Note that the Least pth design provides a lower maximum error at the expense of higher errors at other frequencies. The weighted objective function provided a maximum error of only 0.37 dB but the error is higher at high frequencies compared to the least-squares design.

6. CONCLUSIONS

The design of recursive filters can be greatly simplified by using a standard spreadsheet program. By using a built in optimization capability in a spreadsheet program the optimal design of a stable digital filter can be accomplished without writing any custom code. Since the program is fully interactive, a designer can monitor the progress of the optimization by examining the response curves during the optimization runs.

It has been demonstrated that a weighted least-squares objective function with the logarithmic error in dB provides a very good design of a recursive digital filter. The inclusion of a weighting function allows the minimization of error in any range of frequencies. The use of a logarithmic error function which is the difference in gain in dB allows a better match of gains at frequencies where the desired gain is small.

The Least pth design method can be easily implemented in a spreadsheet form. As demonstrated via the example above, the Least pth design will give a minimax solution to the design problem. The penalty incurred is that minimizing the error at one frequency can increase the error at another. A weighting function can help to distribute the errors as desired.

The techniques described here can be easily ex-

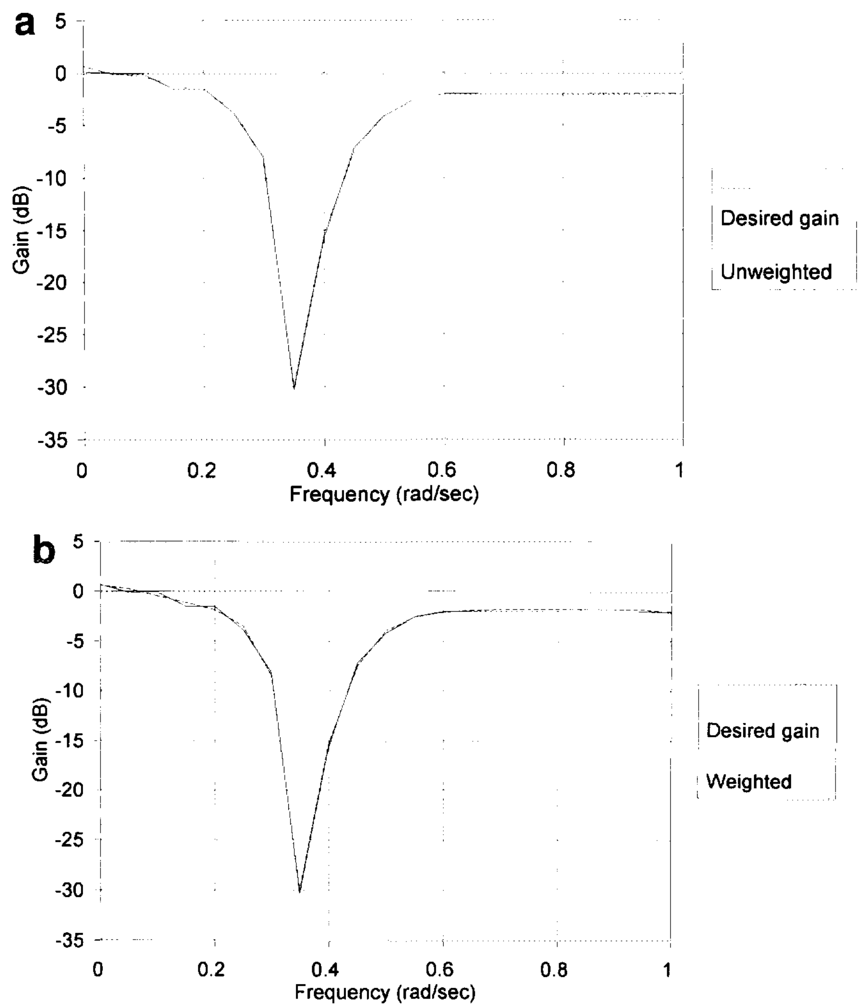

FIG. 4. Magnitude frequency response for Least *p*th design (a) without weighting and (b) with weighting.

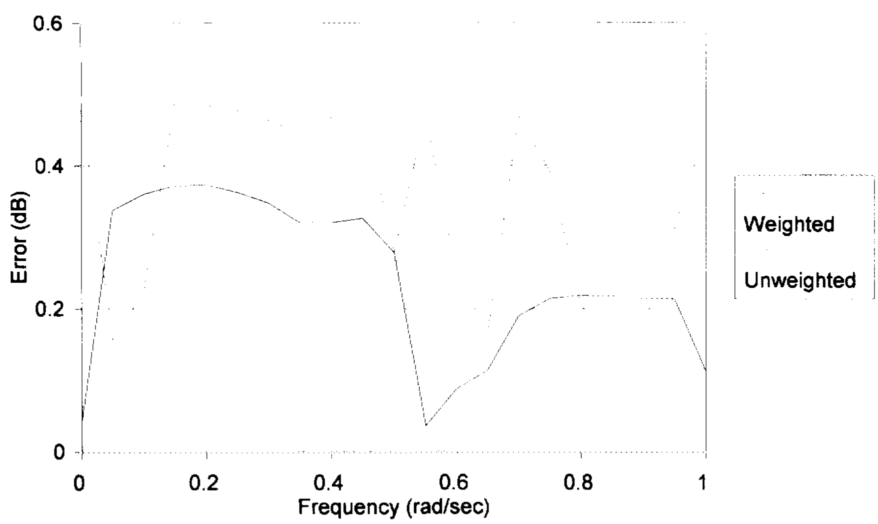

FIG. 5. Error response for least *p*th design with and without weighting.

tended to system identification. Given a frequency response for a discrete time sampled data system it is possible to use the weighted least-squares or least *p*th methods to identify the system transfer function. The optimization procedure can be implemented via a spreadsheet in the same manner as the digital filter design method described above.

Another interesting application for this technique is in the design of recursive delay equalizers. It is possible to define a criterion for the performance of a digital equalizer, which is used to equalize the phase distortion of a digital filter. This criterion can be optimized with respect to the parameters in the equalizer transfer function. Further work will be performed on this problem.

ACKNOWLEDGMENT

The authors acknowledge the support for this research by the Office of Naval Research under program element 060115N.

REFERENCES

1. Antoniou, A. *Digital Filters Analysis, Design and Applications,* 2nd ed., McGraw-Hill, New York, 1993.
2. Steiglitz, K. "Computer Aided Design of Recursive Digital Filters," *IEEE Trans. Audio Electroacoust.* **AU-18** (June 1970), pp. 123–129.
3. Antoniou, A. "Improved Minimax Optimisation Algorithms and Their Application in the Design of Recursive Digital Filters," *IEE Proc.* **138,** G, (December, 1991), pp. 724–730.
4. Franklin, G., Powell, J. D., and Workman, M. L. *Digital Control of Dynamic Systems,* 2nd ed. Addison-Wesley, Reading, MA, 1990.
5. Fletcher, R. *Practical Methods of Optimization,* Unconstrained Optimization, Vol. 1. Wiley, New York, 1980.
6. Charalambous, C. "A Unified Review of Optimization," *IEEE Trans. Microwave Theory Techniques,* **MTT-22** (March 1974), pp. 289–300.
7. Charalambous, C., and Antoniou, A. "Equalisation of Recursive Digital Filters," *IEE Proc.,* Vol. 127, G, October, 1980. pp. 219–225.
8. Charalambous, C. "Acceleration of the Least *p*th algorithm for Minimax Optimization with Engineering Applications," *Mathematical Programming,* **17** (1979), pp. 270–297.
9. Press, W., Flannery, B. P., Teukolsky, S. A., and Vetterling, W. T. *Numerical Recipes,* Cambridge Univ. Press, Cambridge, UK, 1986.
10. Sidman, M. D., DeAngelis, F. E., and Verghese, G. C. "Parametric System Identification on Logarithmic Frequency Response Data," *IEEE Trans. Automatic Control,* **36,** No. 9 (September 1991), pp. 1065–1070.

RUSSELL E. TRAHAN, JR. is a native of New Orleans, Louisiana. He was born on January 30, 1949. He received the B.S. and M.S. degrees in engineering sciences from the University of New Orleans in 1970 and 1973, respectively. He received a Ph.D. in electrical engineering from the University of California, Berkeley, in 1977, with a specialization in optimization and control. Since 1977 he has held a faculty position in the Department of Electrical Engineering at the University of New Orleans, and he now has the rank of Chevron USA Professor of Electrical Engineering. In 1980 he returned to the University of California, Berkeley, as a visiting assistant professor. During the summers of 1986 through 1989 he was an ASEE Summer Faculty Research Fellow at the Naval Research Laboratory–Stennis Space Center, MS. His current research interests are in the areas of controls, optimization, and signal processing. Dr. Trahan is a member of the IEEE Control Systems Society and is active in the New Orleans Section of IEEE. He was named the Outstanding Engineering Educator in Region 3 of IEEE in 1991. He is also a member of ASEE, Eta Kappa Nu, Phi Kappa Phi, and Omicron Delta Kappa.

FRANCIS B. GROSZ, JR. is a native of New Orleans, Louisiana. In 1970 he received a B.S. in physics and a B.S. in engineering sciences and in 1973 received the M.S. in engineering all from the University of New Orleans. In 1979 he received a Ph.D. in electrical engineering from the University of Illinois, Urbana–Champaign. He is a registered professional engineer in electrical engineering in Louisiana. He has been employed as a broadcast engineer at WWL-AL/FM/TV in New Orleans, as a principal engineer at Litton Data Systems, as an engineering specialist at Litton DSD's Electro-Optics Research Center, as a member of the electrical-engineering faculty at the University of New Orleans, and as a senior electrical engineer with the Naval Research Laboratory at Stennis Space Center, Mississippi. He is currently with Omni Technologies, Inc., a research and development consulting firm in Metairie, Louisiana. Dr. Grosz served as vice-chairman of the 1990 IEEE International Symposium on Circuits and Systems and on the executive committees for the 1989 IEEE Transmission and Distribution Conference and the 1994 IEEE ICC/Supercomm. He is a member of IEEE and Eta Kappa Nu.

SEAN GRIFFIN was born in New Orleans, Louisiana, on June 11, 1965. He received a B.S. in electrical engineering from Louisiana State University in 1987. He has worked for General Dynamics designing avionics systems for the Advanced Tactical Fighter and has worked for Litton Data Systems on fiber optic sensor projects. Mr. Griffin worked for the Naval Research Laboratory as an electrical engineer for four years, where he developed marine electronics systems. He is currently vice president of Omni Technologies, Inc., which is a research and development firm. His research interests include sensor design and characterization, signal processing, and communications.